X-Ray Physics

Evan Berkowitz*
Junior, MIT Department of Physics
(Dated: October 25, 2006)

We measure a variety of phenomena related to X-Ray absorption and production. We present data which conforms within reasonable error to Moseley's Law for spectral lines. Our experiment also conforms within experimental error to the approximation that doublet splitting goes as $(Z\text{-}\sigma)^4$. Bremsstrahlung spectra are presented and analyzed, as are ^{22}Na electron-positron annhilation lines. The importance of understanding x-rays is demonstrated by a brief overview of their impact on physics at the time of their discovery.

1. INTRODUCTION AND STATEMENT OF PROBLEM

X-Rays were discovered by Wilhelm Conrad Röntgen in November of 1895 through experimentation with cathode-ray tubes. Rntgen noticed that with enough voltage supplied to the tube, a flourescent screen lit up even though it was shielded from the cathode-ray tube by heavy black cardboard that should have prevented electromagnetic radiation from reaching the screen. After placing other matter before the screen and still observing the flourescence, he placed his hand before the screen and was able to see the shadow of his bones on the screen.[1] Within a month of his discovery, the phenomenon was known the world over.[2]

Röntgen's x-rays shocked the scientific world because it was the first time matter-penetrating radiation had been observed. Röntgen himself was so fearful for his scientific reputation that he did not submit his results for over two months, to allow himself time to ensure repeatability and accuracy and to eliminate his own doubts. This new phenomenon, while not understood, had practical application immediately. In January, his paper is published, and by February of the same year x-ray photography was used to identify the site of required surgery. By 1899, x-rays were used as a treatment for cancer.[3]

In 1913 Henry Moseley, one of Rutherford's graduate students, performed a systematic study of 38 target elements as the anode in an experimental setup similar to Röntgen's, measuring x-ray spectra of elements between aluminum and gold. After data analysis, Moseley discovered that the peaks in the energy spectra emitted by the various elements corresponded to the atomic number squared. He stated:

> We have here a proof that there is in the atom a fundamental quantity, which increases by regular steps as we pass from element to the next. This quantity can only be the charge on the central positive nucleus, of the existence of which we already have definite proof... We are therefore led by experiment to the view that N [the atomic number] is the same as the number of the place occupied by the element in the periodic system.[4]

Moseley's law allows for the experimental determination of atomic number, and corrects Mendeleev, who had assumed that periodicity was determined by atomic mass, even though some pairs of elements violated this order (e.g. K_{19} has atomic mass 39.09 while $_{18}$Ar has atomic mass of 39.95). It also allows for prediction of the number of elements missing from the table. For instance, La (mass 138.9) was discovered in 1839 and Lu (mass 174.9) in 1907, but no one could say with certainty how many elements lay between them. Once the atomic numbers were determined to be 57 and 71 respectively, the existence of fourteen lanthanides is required. One can even determine chemical identities by matching an unknown's spectrum to known spectra.

It is now understood that an x-ray is a high-energy electromagnetic wave. Only after understanding their powerful nature can we understand the importance of properly practicing x-ray safety by limiting direct exposure.

Clearly, understanding x-ray spectra is important relevent for atomic physics and can give insight into various physical properties. It is important, then, that we understand what we see when we measure spectra from similar experimental setups. I present a variety of spectra and analyze them to see their conformance with Moseley's law. I also show other spectra which can be observed with similar setups: bremsstrahlung, which requires the deceleration of a charged part when it hits matter. We can calibrate our detector using known values for some lines. One such line is ^{22}Na's 511 keV line, which is a result of electron-positron annihilation.

2. THEORY

2.1. Moseley's Law

Moseley's law states that the energy of a spectral line is related to the difference in energy levels as well as the square of the atomic number, which we denote Z. However, when the square root of the energy is plotted against Z, not all of the lines go through the origin. In

*Electronic address: evan_b@mit.edu

order to accomodate this fact, Moseley made E depend on $(Z-\sigma)^2$ instead of Z alone. Moseley's law states

$$E \;=\; R\left(\frac{1}{{n_f}^2} \;-\; \frac{1}{{n_i}^2}\right)\;(Z-\sigma)^2 \qquad (1)$$

where R is the Rydberg constant in eV, n_f and n_i are the final and initial energy levels respectively, Z is the atomic number, and σ is the shielding. This law can be derived from the Bohr model of the atom, which says that the energy of each level goes as $\frac{1}{n^2}$. For our purposes, it will be sufficient to clump the leading constants together and state the law as

$$\sqrt{E} \;= C_n\;(Z-\sigma) \qquad (2)$$

Before moving on, we should develop an understanding of the physical interpretation of σ. We identify σ to be the "screening factor", with which the other electrons shield the electron which is falling down the energy levels from the atomic charge. For example, if an K-shell electron is expelled by an x-ray, there is still one electron between the nucleus and an L-level electron. This K-shell electron effectively reduces the charge that the L-level electrons feel from the nucleus by contributing adding a negative to the charge between the two.

2.2. Theory of Bremsstrahlung

Electromagnetic theory dictates that an accelerating charge gives off a photon. Bremsstrahlung is the spectrum we observe due to this effect. Incident electrons head towards matter and get deflected. If the electron just grazes an atom, it will only be slightly deflected, and hence will keep most of it's energy, meaning that the photon emimtted will be low energy. The electron can graze the atom closer and closer until it strikes the atom head-on, in which case it can be captured and give up all of its kinetic energy to the photon. We predict, then, that in the bremsstrahlung spectrum we see an high-end energy cutoff, above which we see no photons. This sharp cutoff is difficult to determine experimentally due to a varying detector efficiency which has less stopping power for higher-energy photons as well as the fact that the spectrum we observe is the combination of more than one species undergoing this acceleration.

3. EXPERIMENTAL SETUP

Our experimental design was a simple one, with a very straightforward signal chain, and is shown in Figure 1. When an electron was hit by an x-ray with enough energy, it was ejected from its atom, and pulled down the detector by the bias voltage. This voltage supplied the electron with additional energy so that it could collide with other electrons and free them from their atoms as well. In order to keep noise low, we cooled the detector

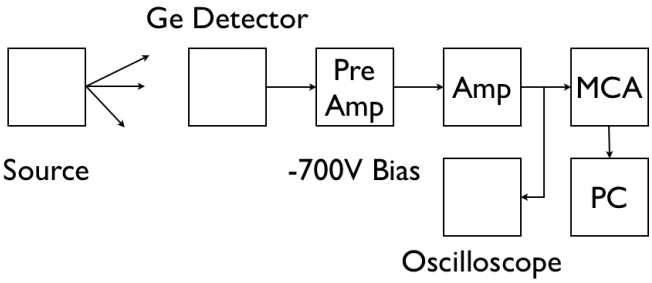

FIG. 1: This is a schematic of the main apparatus and setup of our experiment. The oscilloscope was used during setup to ensure each piece of equipement was functioning properly as it was added. For the different experiments, we varied the source of the radiation. For example, when we measured ^{22}Na's line, we simply put a sample of the material in front of the detector. When we measured the bremsstrahlung of ^{90}Sr, we had a source of Sr emitting β^- at another material, which decellerated the electron, and gave off photons.

with liquid nitrogen, meaning that fewer electrons would jump out of their atom due to thermal motion. When these electrons reached the end of the detector, they were converted into a signal which was fed into a pre-amplifier and then into an amplifier. The amplifier was connected to a multi-channel analyzer (MCA) and then into a PC where we could view the spectrum graphically and perform peak location analysis. We also saved the resulting spectra in order to analyze them in Matlab at a later time. As we set up our apparatus, we connected each component in turn to the oscilloscope to ensure we were receiving the signals we expected. Once this was complete, we left the oscilloscope connected to the amplifier in order to have visual confirmation that we were getting information sent to the MCA.

4. SODIUM 22

Sodium 22 decays through β^+ emission by the reaction

$$^{22}_{11}Na \rightarrow ^{22}_{10}Ne \;+\; e^+ + \nu_e \qquad (3)$$

We see two tall, sharp peaks in the spectrum, as shown in Figure 2. The first peak corresponds to the positron emitted in the β^+ decay being annihilated by an electron in the ^{22}Na sample. This annihilation converts all of the energy of the two particles into two photons of equal energy. Because the electron mass is 511 keV, this first tall peak is located at 511 keV. We also see a second peak which is emitted by the neon nucleus as it falls into its ground-state energy. This transition energy is 1270 keV. Knowing these lines and identifying them reliably allow us to use their values as an energy calibration in later experiments. The 511 keV line appears at bin 1022 and the 1270 keV line appears at bin 2542. Both of these peaks have a FWHM of 5 bins.

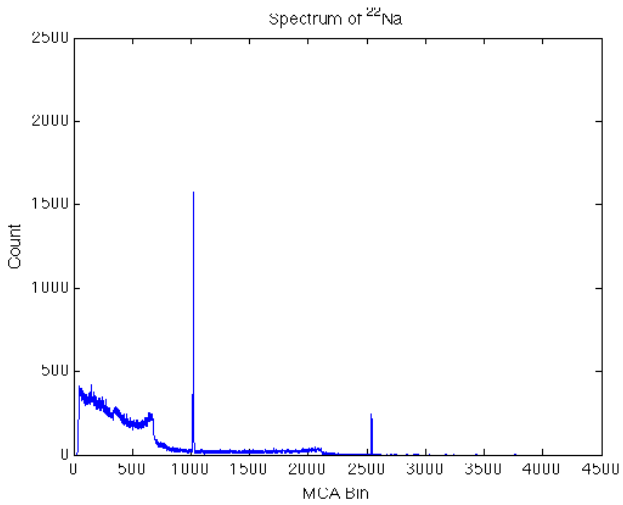

FIG. 2: This spectrum was measured by placing a ^{22}Na sample directly in front of our detector. Two sharp peaks can be seen, one corresponding to electron-positron annihilation and one corresponding to the nucleus of ^{22}Ne falling to its ground state.

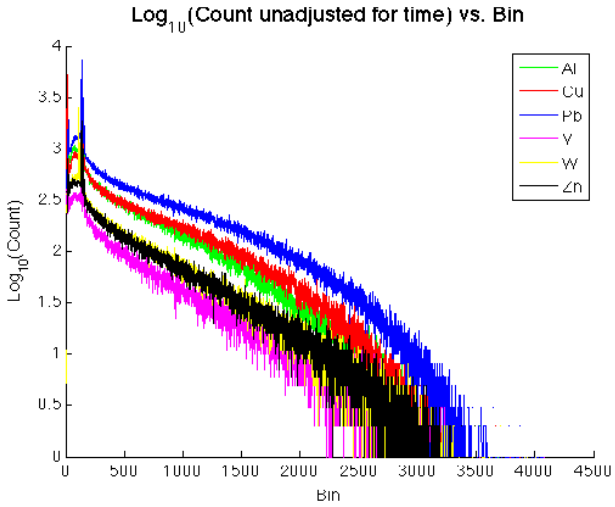

FIG. 3: This plot eliminates the bins with only one count. We plot the separate trials without adjusting for time because otherwise the graphs overlap too much and you cannot see that all the trials join together by dying at about bin 3000.

We also saw the Compton edge at bins 680 and 2125. Compton edges are high-energy cutoff values for the energy that a photon relinquishes when it collides with another particle. This happens when the photon hits it's target electron (for example) and is scattered back in the direction from which it came. With this trajectory, the target electron gains the most momentum, and hence the largest energy available to it.

We cannot use these bin numbers for calibration for each experiment, due to different amplification settings, but identifying the phenomena we see in this spectrum will allow us to confidently calibrate later portions.

5. BREMSSTRAHLUNG

As noted earlier, due to the overlap of more than one bremsstrahlung spectrum as well as a decreased capacity to detect higher-energy photons, getting precise data for the high-energy cutoff will be difficult. Initially, we calibrated our MCA with a ^{22}Na spectrum, observing the 511 keV line at bin 922. Because the MCA recorded only 4096 channels, the maximum energy we could observe is 2270 keV. To study the bremsstrahlung spectrum, we had a ^{90}Sr emit a beam of electrons at a target element. The target element was placed at 45° angle with respect to both the strontium and the detector.

The overlap of two bremsstrahlung spectra make determining an exact value for the strontium cutoff extremely difficult. Strontium decays by β^- into an electron, antineutrino, and yttrium, which immediately decays again into an additional electron and antineutrino as well as zirconium. So, for every electron in the ^{90}Sr spectrum, we also see an electron in the ^{90}Y spectrum. These two

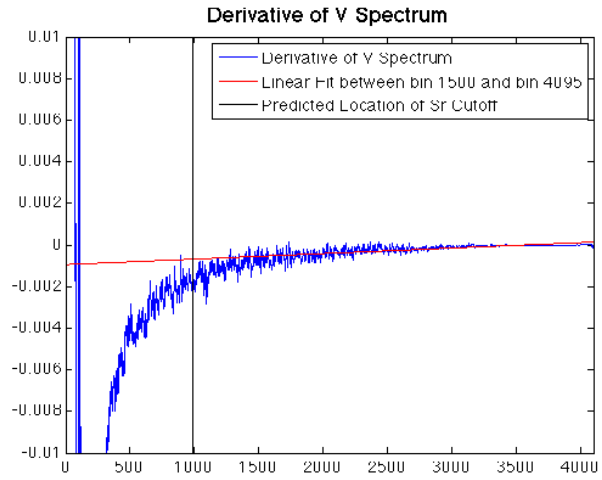

FIG. 4: The derivative of Vanadium's smoothed spectrum. It is representative of the other elements as well, in that all of the spectra's derivatives begin to noticably depart from the linearization of the small portion near bin 1000. Smoothing was performed in Matlab using the function 'smooth'.

electrons are emitted with different characteristic energies: the Sr emits the electron with 546 keV maximum and Y emits the electron with a maximum energy of 2282 keV.[5] These correspond to bins 985 and 4117, the latter of which is off of our scale. So, we expect to see at least some non-noise information all the way down our axis. Additionally, these energy maximums depend only on the ^{90}Sr and ^{90}Y, not on the target elements. We tested a variety of elements to confirm this point.

Many different algorithms were run on the spectra in order to attempt to see an accurate numerical cutoff.

However, due to the extremely wide spectrum of the yttrium electron, we could not find its cutoff, which we expect to be off of the scale. It is possible that if we had taken a much longer sample, we would have seen the end of the spectrum fill out, but by taking the log of the count, we eliminate the 1-count bins and can see more clearly where the more significant signal dies off. We find this cutoff to be centered around bin 3000, as can be seen in Figure 3.

We did have success at geting a rough idea of where the strontium bremsstrahlung cutoff is. We smoothed the original spectra, took their derivatives, and smoothed again (with a much smaller interval). We then noticed that the majority of this plot appeared to be a straight line close to the axis, when it then suddenly departs from the line. We hypothesized that the flat portion of the plot was dominated by the yttrium spectrum whereas the part which departs significantly was dominated by the strontium spectrum. As can be seen in Figure 4, the plot starts to differ from the linear approximation near bin 1000, as expected.

6. MOSLEY'S LAW

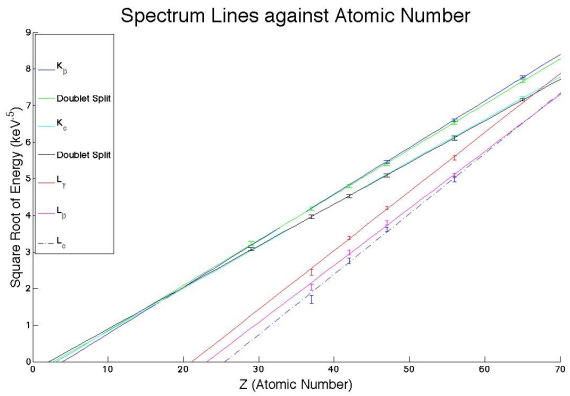

FIG. 5: When the square-root of the energy is plotted against Z, a linear relationship becomes apparent. The average χ^2 value for the K series was 1.6±0.8. For the L series, the average χ^2 value was 4.4±1.1 The errorbars correspond to 1σ, which was determined from the measured FWHM of each peak.

Our apparatus for testing Moseley's law consisted of the detector and signal chain as described above, with an Amersham variable x-ray source. The source contained ^{241}Am, which is radioactive (Americium is transuranic). The source also had a wheel with copper, rubidium, molybdenum, silver, barium, and terbium. To perform the experiment, we chose a setting on the wheel and let the detector process the results of the radiation passing into the target. To calibrate our measurements, we did not use the ^{22}Na line, because it is on an energy scale which is much greater than the scale which we were measuring. Instead, we used two well-known peaks of lead. However, this introduced a systematic error, because the peaks had FWHM of approximately 20 bins, which is significantly larger than the error for the ^{22}Na. The conversion factor from bins to energy only changes in the third decimal place when taking this error into consideration, and hence is most likely not a factor in our measurements.

As seen in Figure 5, when the square-root of the energy is plotted against the atomic number, a clear linear relationship is shown. Using the fitlin function, we get linear fits of the form $E = a(2)Z + a(1)$ instead of the more familiar form $E = C_n(Z - \sigma)$.[6] By factoring, we see that $C_n = a(2)$ and $\sigma = -a(1)/a(2)$. For the K lines, the program then gives us $\sigma = 3.0 \pm 0.5$ and $C_n = 0.12 \pm 0.01$. For the L lines, the program yields $\sigma = 23 \pm 2$ and $a(2) = 0.16 \pm 0.02$. According to Professor Sadoway, the accepted values for σ are 1 and 7.4, approximately.[4] Our measured values are both three times larger, suggesting a missed conversion or some sort of multiplicative systematic error.

In theory, the relative intensities of the K_{α_1} and K_{α_2} should be 2:1.[7] The only measurement we were able to make where the two lines did not have some ambiguous area which they might share was Terbium. For Terbium, the count for α_1 was 9999±100 and the count for α_2 was 5699±75. The ratio of these two values is 1.75±0.04, which is not a fantastic match.

7. CONCLUSIONS

Moseley's Law fit our data, larger sigmas than expected, but by a constant systematic multiplicative factor, revealing some Because the plot was linear, the relationship between energy and atomic number must be quadratic, which is what Moseley's Law states. Our experiment revealed some of the fine-structure of atomic structure by showing some of the doublet splitting for K_α and K_β. We were not able to confirm the prediction of ratios of the intensities of K_α and K_β, receiving a ratio that was too small by more than a few σ. We were able to observe the electron-positron annihilation line and the line resulting from the ^{22}Ne nuclues transitioning into its ground state, as well as the Compton edge for ^{22}Na. We developed some tools which help in analysis of the bremsstrahlung, and were able to estimate the cutoff energy for strontium by examining the derivatives of the spectra. If we had had a much longer trial time we would have been able to gather statistically significant data for the energy cutoff values. Although we had some difficulty with the analysis of our results, we have shown the power of x-rays and their revealing nature.

[1] W. Authors, *Wilhelm Conrad Roentgen*.
[2] MIT Department of Physics, *X-ray Physics*, JLExp31.pdf (2006).
[3] D. R. Sadoway, *Lecture 16 - Characterization of Atomic Structure: The Generation of X-Rays and Moseley's Law*, MIT OpenCourseWare (2004).
[4] D. R. Sadoway, *Lecture 17 - X-ray Spectra, Bragg's Law*, MIT OpenCourseWare (2004).
[5] N. E. Holden, *Table of the Isotopes*, http://www.hbcpnetbase.com/articles/11_02_87.pdf (1987).
[6] S. Sewell, *fitlin.m*, http://web.mit.edu/8.13/matlab/ (2006).
[7] A. Compton and S. Allison, *The interpretation of x-ray spectra* (1935), https://web.mit.edu/8.13/8.13a/references-spring/xrays/x-rays-in-theory-and-experiment-by-compton-and-allison-p590.pdf.