Photonic Crystals and Their Various Applications

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What is a Photonic Crystal?

A periodic structure of dielectric medium on a wavelength scale is called **Photonic Crystal (PC)** or **Photonic Bandgap Material (PBG)**.
Perfect Photonic Band Gap Materials

* Periodic along one direction, and extends infinity along other directions.

* Periodic along two axis, and extends infinity along the third axis.

* Periodic along three axes (x, y, z) and can be obtained by filling (eg. sphere, bar) a unit cell of any three dimensional lattice and duplicating through space

PERFECT → INFINITY (not a slab with a finite height)
The idea of Photonic Crystals was first introduced by Yablonovitch\(^1\) and John\(^2\)

The idea of a Photonic Crystal is based on drawing analogies between light and electrons:

Because both have a wave-like nature and can therefore be diffracted!
Consider Electrons and the Electronic Bandgap first
The electronic bandgap of an insulator arises from the diffractive interaction of the electron wavefunction with the atomic lattice, resulting in destructive interference at certain wavelengths.
What about Photons (ie, Light) and Origin of the Photonic Bandgap?
Interaction of light with matter

Material’s refractive index (or dielectric constant $\varepsilon$) describes the interaction of light with matter!
Setting up a periodic refractive index (like a periodic potential of an atomic lattice) can result in a similar ‘band theory’ for photons where certain frequencies cannot propagate.

In other words, the photonic equivalent of an insulator!

However,
Electrons and photons are not on the same wavelength scale

|Wavelength of Visible Light: | $\lambda \sim 400 \text{ nm-700 nm}$ |
|Wavelength of Electron:      | $\lambda \sim 0.1 \text{ nm}$ |
To see the diffractive effects, we must make large artificial `atoms’ on the same scale as the wavelength
Computer models for doing the calculations for semiconductors cannot be used for photons!

Schrodinger’s equation governs electrons, but Maxwell’s equations describe the behavior of light.

With photons, one cannot safely neglect polarization the way one can with electrons.
The electromagnetic properties of the photonic crystals are completely determined by the solutions of the macroscopic Maxwell’s equations

\[
\vec{\nabla} \times \left( \frac{1}{\varepsilon(\vec{r})} \vec{\nabla} \times \vec{H}_\omega(\vec{r}) \right) = \left( \frac{\omega}{c} \right)^2 \vec{H}_\omega(\vec{r})
\]

\(\varepsilon(\vec{r})\) is the spatially periodic dielectric function that describes the crystal

\[
\vec{\nabla} \cdot \vec{H}_\omega(\vec{r}) \equiv 0 \quad \text{(Transversality condition)}
\]

\[
\vec{H}_\omega(\vec{r}) = e^{ik \cdot \vec{r}} \vec{H}_{\omega,k}(\vec{r}) \quad \text{(Bloch-Floquet theorem)}
\]

Periodicity of \(\varepsilon(r)\) is described by the Bravias lattice associated with the crystal.
\[ \tilde{H}_\omega (\vec{r}) = e^{i\vec{k} \cdot \vec{r}} \tilde{H}_{\omega, \vec{k}} (\vec{r}) \]

\( \vec{k} \) : Bloch vector (crystal momentum)
\[ \vec{H}_\omega (\vec{r}) = e^{i\vec{k} \cdot \vec{r}} \vec{H}_{\omega,k} (\vec{r}) \]

\( \vec{H}_{\omega,k} (\vec{r}) \) denotes the lattice periodic part of the Bloch function, i.e.,

\[ \vec{H}_{\omega,k} (\vec{r} + \vec{R}) = \vec{H}_{\omega,k} (\vec{r}) \]

for all lattice vectors \( \vec{R} \).
Restricting the Bloch vector to the first BZ, corresponds to a back-folding of the dispersion relation in the infinitely extended $k$-space into the 1$^{st}$ BZ by means of translations through reciprocal lattice vectors.

This introduces a discrete band index $n \in \mathbb{N}$ such that the band structure is described by the set

$$\left\{ \left[ \omega_n (\vec{k}), \vec{H}^n_{\omega, \vec{k}} \right] \right\}, \ \vec{k} \in 1^{st} \text{ BZ}, \ n \in \mathbb{N}$$

associated with

$$\vec{\nabla} \times \left( \frac{1}{\varepsilon (\vec{r})} \vec{\nabla} \times \vec{H}_{\omega} (\vec{r}) \right) = \left( \frac{\omega}{c} \right)^2 \vec{H}_{\omega} (\vec{r})$$
\[ \vec{\nabla} \times \left( \frac{1}{\varepsilon(\vec{r})} \vec{\nabla} \times \vec{H}_\omega(\vec{r}) \right) = \left( \frac{\omega}{c} \right)^2 \vec{H}_\omega(\vec{r}) \]

\[ \vec{H}_\omega(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} \vec{H}_{\omega,\vec{k}}(\vec{r}) \]

\[ \vec{H}_{\omega,\vec{k}}(\vec{r}) = \sum_{\vec{G} \in \text{1st BZ}} \sum_{\lambda=1,2} \vec{e}_\lambda,\vec{q} \cdot \vec{q} = 0, \quad \lambda = 1,2 \quad \text{(transverse unit vectors)} \]

\[ \text{(expansion coefficients - to be determined...)} \]
For the sake of simplicity, if we assume the structure itself the same with its reciprocal lattice \((a \rightarrow 2\pi/a)\), we would describe the irreducible Brillouin zone with the half of the unit cell \([0, \pi/a]\) shown in the figure.
One Dimensional Photonic Crystal

GaAs/Air Multilayer Film
alternating layers of width $0.5a$

PHOTONIC BAND GAP

air band
$(0.5, 0.25)$
$(0.5, 0.15)$

dielectric band

$(\omega a/2\pi c)$

$(ka/2\pi)$
\( n_h = 1.52 \)
\( n_L = 1.38 \)
(square lattice of dielectric rods, $\varepsilon = 8.9 \quad r = 0.2a$)
Two Dimensional Photonic Crystal
(square lattice of dielectric rods, $\varepsilon = 8.9$)

$\omega a/2\pi c$

$ka/2\pi$

TE PHOTONIC BAND

TM PHOTONIC BAND

GAP

TE modes

TM modes
2D Hexagonal lattice structure

(\(\varepsilon = 13, \ r = 0.48a\) )
2D Hexagonal

\[
(\frac{\omega a}{2\pi c})
\]

\[
\epsilon = 13, \ r = 0.48a
\]

COMPLETE PHOTONIC BAND GAP

TE modes

TM modes

\[
(ka/2\pi)
\]
(Joannopoulos’s group / MIT)
Complete bandgap (0.46c/a - 0.56c/a)

f = w/d = 0.28

c/d = 1.414

Face centered tetragonal lattice symmetry

What we did ...
Working Principle

* A 2D PBG Structure for Surface Temperature Mapping
* Based on BB Radiation Characteristics of the Target
\[ I(\lambda, T) = \frac{2\pi \hbar c^2}{\lambda^5 (e^{\hbar c / \lambda kT} - 1)} \]

Intensity

Planck's Function

\[ I(\lambda_i, T) \quad I(\lambda_j, T) \quad I(\lambda_k, T) \]

\[ O(\lambda_i, T) \quad O(\lambda_j, T) \quad O(\lambda_k, T) \]

\[ I(\lambda, T) \rightarrow O(\lambda, T) = S\gamma I(\lambda, T) \]
Proposed PBG Structure: Design Parameters

* 2D Photonic Crystal Slab
* Triangular array (ie, 2D Hexagonal) of air holes
* GaAs: Lossless around 1.55-\(\mu\)m
* \(r/a = 0.3\) (\(a = 0.382\) -\(\mu\)m)
* \(\varepsilon = 11.4\)
* Complete TE band-gap: 0.213-0.280 (\(c/a\))
* Defect radii: 0.51\(a\), 0.54\(a\), 0.57\(a\).
Radiation Guiding EM Modes through the waveguide Trapping by the corresponding point defects Obtaining the Scaled Intensities

Obtaining temperature using BBRC and PC transmission response
Relation between measured optical power and resonant wavelengths (output radiation and BBR are linked)

\[ P_{vi}(\lambda_i, T) = S \gamma_i \int_{\lambda_i - \Delta \lambda_i / 2}^{\lambda_i + \Delta \lambda_i / 2} I(\lambda, T) d\lambda \]

\[ P_{vi}(\lambda_i, T) = \int_{\lambda_i - \Delta \lambda_i / 2}^{\lambda_i + \Delta \lambda_i / 2} \frac{dP_v}{d\lambda} d\lambda - \int_{\lambda_i}^{\lambda_i + \Delta \lambda_i} \frac{dP_v}{d\lambda} d\lambda \]

Ratio of optical powers for any two defects \(i\) and \(j\)

\[ \frac{P_{vi}(\lambda_i, T)}{P_{vj}(\lambda_j, T)} = \frac{\gamma_i}{\gamma_j} \int I(\lambda, T) d\lambda \]

Discrete formulisation for numerical analysis

\[ \sum_k^n \left( \frac{\delta P_{vik}}{\delta \lambda_{ik}} \right) \delta \lambda_{ik} = \frac{\gamma_i}{\gamma_j} \sum_k^n \left[ \frac{1}{\exp(hc / \lambda_{ik} T - 1)} \right] \delta \lambda_{ik} \]

\[ \sum_l^m \left( \frac{\delta P_{vjl}}{\delta \lambda_{jl}} \right) \delta \lambda_{jl} = \frac{\gamma_i}{\gamma_j} \sum_l^m \left[ \frac{1}{\exp(hc / \lambda_{jl} T - 1)} \right] \delta \lambda_{jl} \]
\[ \lambda_{ik} \equiv \lambda_i - \Delta \lambda_i / 2 + (k - 1/2)\delta \lambda_i \]
\[ \lambda_{jl} \equiv \lambda_j - \Delta \lambda_j / 2 + (l - 1/2)\delta \lambda_j \]
\[ (n-1)\delta \lambda_i = \Delta \lambda_i \]

\[
\left\{ \begin{align*}
\sum_k^n \delta p_{vik} - \frac{\gamma_i \sum_k \left[ 1 / \exp(hc/\lambda_k T - 1) \right] \delta \lambda_i}{\gamma_j \sum_l \left[ 1 / \exp(hc/\lambda_l T - 1) \right] \delta \lambda_j} & \equiv f(T) \\
\end{align*} \right\}
\]

\[
\lim_{T \to T'} f(T) = 0
\]

\[
S = X\Theta \prod R
\]
Our Photonic Crystal Structure

\( (\omega a/2\pi c) \)

\( (\varepsilon = 11.4, \ r = 0.3a) \)

TE Photonic Band Gap

\( (ka/2\pi) \)
(A line defect is introduced in the periodic array)
Line defect is introduced in the periodic array...

\[
\frac{\omega a}{2\pi c} = 0.30, 0.25, 0.20, 0.15, 0.10, 0.05
\]

\[
\frac{ka}{2\pi} = 0.20, 0.30, 0.40, 0.50
\]

\[
(\varepsilon = 11.4, r = 0.3a)
\]

TE modes

TE Photonic Band Gap

waveguide modes
... and point defects are introduced.

\[ (\omega a/2\pi c) \]

\[ (ka/2\pi) \]

waveguide modes

TE modes

\( \lambda_1 \)

\( \lambda_2 \)

\( \lambda_3 \)
Radiation

Guiding EM Modes through the waveguide

Obtaining the Scaled Intensities

Trapping by the corresponding point defects

Obtaining temperature using BBRC and PC transmission response
Defect Size: $r = 0.51a$

Surface plot for the intensity of $H_y$ component of EM field for the defect radius of $0.51a$, corresponding to the resonant wavelength of $1.73 \, \mu m$.

Amplitude of the $H_y$ component of the field in a.u. for the defect, $0.51a$. The each colored line indicates one slice passing through the isolated point defect.
Surface plot for the intensity of $H_y$ component of EM field for the defect radius of 0.54$a$, corresponding to the resonant wavelength of 1.699 µm.

Amplitude of the $H_y$ component of the field in a.u. for the defect, 0.54$a$. The each colored line indicates one slice passing through the isolated point defect.
Defect size: $r = 57a$

Surface plot for the intensity of $H_y$ component of EM field for the defect radius of $0.57a$, corresponding to the resonant wavelength of 1.658 µm.

Amplitude of the $H_y$ component of the field in a.u. for the defect, $0.57a$. The each colored line indicates one slice passing through the isolated point defect.
<table>
<thead>
<tr>
<th>λ (μm)</th>
<th>r (a)</th>
<th>Coup. (Γ)</th>
<th>Inp. Rad. (W/cm².μm)</th>
<th>Out. Rad. (W/cm².μm).S</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.730</td>
<td>0.51</td>
<td>0.95</td>
<td>28.500</td>
<td>27.075</td>
</tr>
<tr>
<td>1.699</td>
<td>0.54</td>
<td>1.00</td>
<td>28.775</td>
<td>28.775</td>
</tr>
<tr>
<td>1.658</td>
<td>0.57</td>
<td>0.77</td>
<td>29.040</td>
<td>22.361</td>
</tr>
<tr>
<td>1.584</td>
<td>0.60</td>
<td>0.49</td>
<td>29.310</td>
<td>14.362</td>
</tr>
</tbody>
</table>

Calculated input and output radiations and coupling ratios, corresponding to the resonant wavelengths in the first column, are illustrated. Second column gives the corresponding defect radii, while the third column indicates the relative power coupling for these defects, which is calculated by the FDTD method, in terms of a constant Γ. In the fourth and fifth columns are given the input radiation in BB radiation form and output radiation, which is emitted from the crystal structure, respectively. Note that the output radiation has a constant scaling factor S, which our method for temperature reading makes use of. Blue colour (last row) indicates the fourth defect, which has a different profile than the others.
SOME OTHER PC APPLICATIONS
Ultra-small optical integrated circuits by 3D Photonic Crystals

Ultra-small multiwavelength light source

ultra-small wavelength DEMUX circuit

(S Noda, Kyoto University, Japan)
Photonic Bandgap Lasers on InP Substrates

* A small defect (ring with no holes) is introduced in pattern.
* Light is trapped in the ring defect and generates laser oscillations.
* World’s smallest laser.

(Yokohama National University/Baba Research Lab)
2D PBG Laser

H Park et al, APL79 3032(2001)
Photonic Crystal Waveguide

High Density Multi-layer PBG Interconnects

(University of Delaware)
Crosstalk Reduction
Using Photonic Crystal Resonators

(Johnson et al, Optics Letters, Dec 1998)
Tapped Delay Line Filter
RF Signal Modulated on Optical Carrier

~ 50 cm² of Wafer Area for every 1 us of Delay

(MIT Lincoln Laboratory)
New Ways to Guide Light