PHYS 311 FALL 2015 HWA 3. Due Wed. Oct. 28'st.

Aspects of Four-Momentum and its conservation

1. Consider a particle of mass $m > 0$. Show that there exists a physical inertial frame in which the four-momentum of this particle has only the zeroth component; that is it has only energy not spatial momentum. Hint: First consider rotating to an inertial frame for which the momentum is along its $\hat{\imath}$ direction and then use one dimensional Lorentz transformations.

2. Consider a collection of N particles of masses $m_i \geq 0$. They each have a four-momentum that we describe as $P_i := (E_i/c, \vec{P}_i)$ and as a consequence one has $P_i \circ P_i = m_i^2 c^2$; their Lorentz norm is constant and invariant. Now construct the total four-momentum of the system as $P_{\text{TOT}} = \sum_i P_i$. Show that $P_{\text{TOT}} \circ P_{\text{TOT}} \geq 0$ and that the equality applies when there are only massless particles that travel along the same direction. So barring this last possibility one can always go to an inertial frame in which the total four-momentum has vanishing spatial components. The zeroth component in this frame, which is nothing but the total energy, is called the *invariant mass* of the system.

3. Prove that a particle A of mass $m > 0$ can not undergo a process $A \to A + \gamma$, where γ represents a massless particle. Can it do $A \to \gamma + \gamma'$?

4. Consider a massless particle in an inertial frame S, having momentum $\vec{P} = p\hat{\imath}$ with $p > 0$, so that it has energy pc. Now consider another inertial frame S' that has a relative velocity $\vec{v} = \beta c \hat{\imath}$. Find E' and $\vec{P}'$ in this frame using Lorentz transformations. What happens to them when $\beta > 0$ and $\beta < 0$? Now simply borrow the equation $E = hf$ from quantum physics which states that the energy is Planck's constant times the frequency of oscillations for a photon. What physical phenomena you observe?

5. Consider a particle of mass m which sits at the center of the co-ordinate system for an inertial observer S. In this frame a photon of energy E -surely $E > 0$- is incident to the particle from the left with $\vec{p} = E/c\hat{\imath}$. Assume after the collision the objects move along the x axis and find the final momenta and energy for both. As a special case consider the case for $m \to \infty$ how does the photon behave in this case? Hint: Note for instance, that if a photon has momentum $p'\hat{\imath}$ with p' unknown the photon energy is $E' = |p'|c$.

6. Assume a spaceship is equipped with a laser gun that emits photons of energy E'_o. This energy is surely measured in the inertial frame of reference of the ship. Now assume that this ship is moving with velocity $\vec{v} = \beta c \hat{\imath}$ with $\beta > 0$ in relation to an observer S. What is the energy of the photon in the frame S.

7. Consider the situation above with the addition of an infinitely massive mirror that is at rest in the frame of S. What is the energy E'_s of the photon that is scattered back from the mirror in the frame of the ship. Is it true that $E'_s > E'_o$ so would you fire to a mirror while running towards it? Here is another question about laser guns: We know that water refracts

light. So if you are trying to fish with a spear you have to take this effect into account since the spear is assumed to be a straight object. Now assume you have a laser gun that refracts like ordinary light. How do you shoot at the fish?

8. Consider a spaceship that is equipped with side laser guns. That is they are aimed in the orthogonal direction in relation to the front of the ship. They emit photons of energy E'_o. Now assume that this ship has velocity $\vec{v} = \beta c\hat{\imath}$ with $\beta > 0$ in another inertial frame S; that is it moves along the end-front axis of the ship. Find the energy of the emitted photons in S. Find also the direction of the photon's velocity in the frame S assuming it uses the same unit directions as S'.

Extra problem: This problem is not mandatory to get full score for this homework but it will earn you extra points if you solve it properly.

Assume there is a constant force field given as $\vec{F} = -f\hat{\jmath}$ with $f > 0$ in the band defined by $0 \leq x \leq L$. It is such a field that it also influences massless particles. That is the equations $d\vec{P}/dt = \vec{F}$ and $dE/dt = \vec{F} \cdot \vec{u}$ are still valid with the constraint that we have a massless particle which obeys $E = |\vec{P}|c$ and $\vec{u} = c^2\vec{P}/E = c\hat{u}$. Now consider a massless object with momentum $\vec{P} = p\hat{\imath}$ with $p > 0$ is incident towards this region with the force field. It will exit it in time T. Find the velocity vectors and its drift along the y axis at this time. Also describe the angle of the velocity vector of the photon with respect to the x axis. Hint: Remember that its speed along the x direction is NOT necessarily a constant in the force field, that is force and acceleration are not proportional vectors in special relativity.