PHYS 402 Spring 2017 2'nd homework assignment. Due Feb 28'th.

In what follows we shall assume the validity of the entire Maxwell's equations

$$\begin{aligned} \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} &= 0 \\ \vec{\nabla} \cdot \vec{E} &= \rho/\epsilon_o \\ \vec{\nabla} \times \vec{B} - \mu_o \epsilon_o \frac{\partial \vec{E}}{\partial t} &= \mu_o \vec{J} \end{aligned}$$

Also remember that as a result of these equations one has

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$$

1. Assume that we are in a region where $\vec{J} = 0$ and $\rho = 0$.

   - Show that one has

   $$\begin{aligned} \left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \vec{E} &= 0 \\ \left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \vec{B} &= 0 \end{aligned}$$

   where $c^{-2} = \mu_o \epsilon_o$.

   - Now as with any linear differential equation, superposition principle is in effect and hence we can look for building block solutions. Show that the following provide a solution

   $$\vec{E} = \vec{E}_o(\vec{k}) \ \sin \left[\vec{k} \cdot \vec{r} - wt \right]$$
   $$\vec{B} = \vec{B}_o(\vec{k}) \ \sin \left[\vec{k} \cdot \vec{r} - wt \right]$$

   provided $k^2 = \omega^2 c^2$. Note that $\vec{k}$ labels the solutions.

   - However the original equations must also be satisfied. Using them show that one must have

   $$\begin{aligned} \vec{k} \cdot \vec{E}_o &= 0 \\ \vec{k} \cdot \vec{B}_o &= 0 \\ \vec{k} \times \vec{E}_o - \omega \vec{B}_o &= 0 \\ \vec{k} \times \vec{B}_o + \frac{\omega}{c^2} \vec{E}_o &= 0 \end{aligned}$$

   - First of all show that these give

   $$E_o^2 = c^2 B_o^2$$

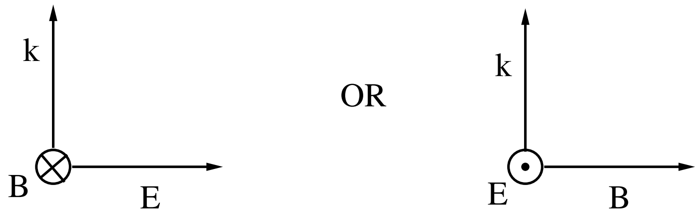
   and

   $$\vec{E}_o \cdot \vec{B}_o = 0$$

   and also

   $$\vec{E}_o \times \vec{B}_o = \frac{E_o^2}{\omega} \vec{k}$$

- Now in view of all these show that the following are the only possibilities



These different possibilities represent another discrete physical degree and is called the polarization of the wave.

2. Solve problem 7.44, 7.47, 7.49.

3. Solve problem 7.64 (doing the necessary reading). Now in view of this exercise which assumes the existance of magnetic monopole, assume every object in the universe have the same electric to magnetic charge ratio. Can you arrive at a theory which is synonymous with Maxwell's electrodynamics -the one where there is no magnetic charges- ?

4. We have derived the Lorentz transformations in lecture. If there is an inertial observer S with co-ordinates x and t and another one S' with co-ordiantes x' and t' and with relative velocity $\vec{v} = v\hat{\imath}$ in relation to S these transformations are

$$\begin{aligned} ct' &= \gamma(ct - \beta x) \\ x' &= \gamma(x - \beta ct) \\ y' &= y \\ z' &= z \end{aligned}$$

   where $\beta = v/c$ and $\gamma = (1 - \beta^2)^{-1/2}$.

   Expand these transformations to first order in v/c and note that there is a linear contribution to the time co-ordinate. Devise an argument as to why this effect was not observed historically earlier. Hint: Observation earlier historically means that the available distances were relativelty small durations were relatively large and speeds were relatively low.

5. Consider one has two perfectly synchronized clocks; one one Earth and one on the moon. When we observe via a telescope the clock on the moon we see that it shows 12:00. What is the actual time reading of the clock at that instant on the moon.

6. Via a powerful telescope we have observed that a large armada of spaceships set out towards earth yesterday. The distance to the planet is one light years. Yet they arrived for invasion the day after the observation. Did they travel faster then light?