PHYS 411 Fall 2014 FINAL. Tue Jan 6'th 2015 13:00-16:00 YD 202_
Absolutely no notes, no electronic gadgets of any form and no questions are allowed. All questions have equal value points. In this exam no formulas are provided. Exam duration is 3 hours.

---

1. Consider a one dimensional system where a particle of mass m is under the influence of a potential $V(x) = -E_o\delta(x)/L$, where all parameters are positive. By integrating the energy eigenvalue equation find the condition on the discontinuity on the first derivative of the wavefunction. Find the bound state energy eigenvalues and eigenfunctions.

2. Consider the potential of the previous question. Assume that a unit current is incident from $x = -\infty$. Find the positive energy eigenstates under this condition. Find also the transmission and reflection coefficients. Hint: Remember the importance of the quantum current.

3. A hydrogenic (single electron) atom of nucleus charge Z has been observed to be in the lowest energy eigenstate. Assume that suddenly the nucleus decays by emitting an alpha particle. What is the probability to observe the new system in the ground state? Hint: The ground state wave-function of a hydrogenic atom is $\psi = N(Z, a)\exp(-Zr/a)$.

4. Remembering $\vec{L} = -i\hbar\vec{r} \times \vec{\nabla}$, show that one has

$$\nabla^2\psi = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) - \frac{L^2}{\hbar^2 r^2}\psi$$

   Hint: It is useful to know that $(\vec{A} \times \vec{B})_i = \epsilon_{ijk}A_jB_k$ and that $\epsilon_{ijk}\epsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{jl}\delta_{im}$.

5. Consider a particle of mass m under the influence of a constant magnetic field $\vec{B}$. Expand the hamiltonian using $\vec{A} = (1/2)\vec{B} \times \vec{r}$. Simplify your result as much as possible using spherical polar co-ordinates and the angular momentum operators. Hint:$(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D}) = (\vec{A}\cdot\vec{C})(\vec{B}\cdot\vec{D}) - (\vec{A}\cdot\vec{D})(\vec{B}\cdot\vec{C})$

6. A particle of mass m is in a cubic box of side length L. Find the energy eigenstates and eigenvalues and emphasize the degeneracies. Now assume that the energy of the system is measured to be the lowest possible value. After this the length of the box along the z direction is suddenly doubled. What is the probability to measure the same energy value as before.