PHYS 532 Spring 2014 1st Homework Assingment. Due Wed Feb 26 at the lecture.

This set of exercises will review some of the notions we have covered last semester on special relativity. We shall not go through everything we did last semester. And, I remember I have said a two-week hwa but I decided to split it anyways. So this one is due next wednesday. Also note that I am generally using a system where $c = 1$ and $\hbar = 1$. It is a good time for you to get used to it.

1- The Lorentz transformations between two inertial observers having a relative velocity $\vec{v} = \beta\hat{\imath}$ are given to be

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$$

where $\gamma = 1/\sqrt{1-\beta^2}$ and also $y' = y$ and $z' = z$. Note that $\beta < 1$ as a requirement. So no physical observer can have relative speeds exceeding the speed of light.

1-1. Show that if you are given two events the time and space separations are not influenced by the transformation above and one has

$$\Delta t'^2 - \Delta x'^2 - \Delta y'^2 - \Delta z'^2 = \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 \equiv \Delta s^2$$

1-2. Classify separations that has $\Delta s^2 > 0$ as time-like, those that has $\Delta s^2 < 0$ as space-like and the ones with $\Delta s^2 = 0$ as time-like. For the following confine the study to one space dimensions for simplicity.

1-2-1. Prove that if two events has a time-like separation there is a *physical* observer where those events occur at the same place.

1-2-2. Prove that if two events has a space-like separation there is a *physical* observer where events occur simultaneously.

1-2-3. From the above argue that occurrence order of events that are time-like separated are observer independent.

1-3. Define $\beta = \tanh(\phi)$. The quantity ϕ is called the rapidity. Describe the Lorentz transformations along the x directions via a matrix $B_x(\phi)$.

1-3-1. Show that $B_x(\phi_1)B_x(\phi_2) = B_x(\phi_1+\phi_2)$. Thus rapidities are additive.

1-3-2. Show that in terms of $B_x(\phi)$, the Lorentz transformations are as if one has a pure imaginary coordinate (time) along with a pure real coordinate (space) and that the transformation is as if we rotate with a pure imaginary angle.

1-4. We know that if an observer measure the velocity of a particle as $\vec{u}$ its energy and its momentum are given as $E = m\gamma(\vec{u})$ and $\vec{p} = m\gamma(\vec{u})\vec{u}$ respectively. Where $\gamma(\vec{u}) = 1/\sqrt{1-u^2}$. Show that $E^2 - p^2 = m^2$ which is an invariant. Note also that the velocity $\vec{u} = \vec{p}/E$ a generally true expression (regardless of $m = 0$ or not)

1-4-1. Since the four-vector p^μ with $p^0 = E$ and $p^1 = p_x$ etc.. has a positive Lorentz norm, it is a time-like four-vector. Thus there must be an observer where the energy is simply m and the momentum is zero. Find this observer's rapidity variable.

1-4-2. One can also represent a particles energy and momenta in terms of a rapidity as $E = m\cosh(\phi)$ and $p = m\sinh(\phi)$. Express $\cosh(\phi/2)$ and $\sinh(\phi/2)$ in terms of E and p.

1-4-3. Using Lorentz transformations of the four-momentum of a massless particle and borrowing the Planck relation $E = f$, find the formula for relativistic Doppler effect.

1-5. Define the object $g_{\mu\nu}$ where $-g_{00} = g_{11} = g_{22} = g_{33} = -1$ and it vanishes otherwise. Find the elements of p_μ from $g_{\mu\nu}p^\nu$ (Einstein's summation convention is understood).

1-6. Show that $(p^0)^2 - (p^1)^2 - (p^2)^2 - (p^3)^2$ can be written as $g_{\mu\nu}p^\mu p^\nu$ and as $p^\mu p_\mu$.

1-7. As Lorentz transformations are relating quantities between different observers they should have the form $p'^\mu = \Lambda^\mu{}_\nu p^\nu$. Find the elements of $\Lambda^\mu{}_\nu$ in terms of the rapidity of two observers under consideration.

1-8. Show that the invariance of Lorentz square for four-vectors $V^\mu V_\mu = V'^\mu V'_\mu$ where $V'^\mu = \Lambda^\mu{}_\nu V^\nu$ results in a relation involving g and Λ. Interpret this relation. Also try to make a connection to rotational transformations which are orthogonal matrices ($O^T O = OO^T = \mathrm{Id}$).

1-9. **Devise a problem** from the concepts used in this assignment set. Do not copy and be creative. There is no limit for complication.