Lepton number violation in top-color assisted technicolor

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We calculate the rates for lepton number violating processes via the exchange of the $Z'$ boson occurring in top–color assisted technicolor. We find that $\mu-e$ conversion in nuclei is about an order of magnitude better than $\mu\rightarrow 3 e$ for constraining the magnitudes of the lepton mixing angles. The decay $\mu\rightarrow e\gamma$ yields much weaker bounds. The current experimental limits allow for a mass of the new gauge boson around 1 TeV and the magnitudes of the mixing angles turn out to lie roughly between the analogous elements of the CKM matrix and its square root. [S0556-2821(99)02709-5]

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I. INTRODUCTION

Observation of lepton number violation would be one of the most spectacular pieces of evidence for deviations from the standard model. Lepton mixing arises naturally in many of the extensions of the standard model. Here we consider the top-color assisted technicolor (TC2) scenario introduced by Hill [1]. In TC2 there exists an extra $U(1)$ group which breaks at a higher energy than the electroweak breaking scale. The couplings of this $U(1)$ are generally not generation universal, so there will be flavor-changing neutral current processes including the possibility for lepton number violation.

After the exposition of the theoretical framework in Sec. II, we present the calculations of the rates for $\mu\rightarrow e\gamma$, $\mu\rightarrow 3 e$, $\mu-e$ conversion in nuclei and the intrinsic dipole moments for leptons in a TC2 scenario for which Chivukula and Terning [2] did a study on constraints from precision $Z$ data.

The decay rate for $\mu\rightarrow e\gamma$ does not provide so stringent a limit since it is a one loop process involving a photon vertex. With the current data, $\mu-e$ conversion in Ti gives limits that are roughly an order of magnitude better than $\mu\rightarrow 3 e$. This will considerably improve with the proposed MECO experiment [3]. For the TC2 scenario we have considered, we found that the current experimental limits allow for mixing angles of magnitude ranging between the analogous elements of Cabibbo-Kobayashi-Masakawa (CKM) matrix $K$ and the elements of $\sqrt{K}$.

II. THEORY

In TC2, one has $SU(2)\times U(1)_1 \times U(1)_2$ as the electroweak gauge symmetry. The extra $U(1)$ will generate a new neutral gauge boson with a mass expected to be around 1 TeV [4]. We require the breaking to occur in two stages, in the following pattern:

$$SU(2) \times U(1)_1 \times U(1)_2 \xrightarrow{U(1)_Y} U(1)_EM$$

Here $Y_1$ and $Y_2$ are the generators of $U(1)_1$ and $U(1)_2$ respectively and $Y=Y_1+Y_2$ is the ordinary hypercharge.

After the first step of symmetry breaking we want the gauge boson corresponding to $U(1)_Y$ to remain massless so that the second stage can proceed as in the standard model. This requires the first stage to be triggered by a neutral, $SU(2)$-singlet condensate. The second stage can be triggered by $SU(2)$-doublet condensates as in the standard model.

The covariant derivative for $SU(2)\times U(1)_1 \times U(1)_2$ is $\partial^\mu + ig T^a W^\mu_w + ig' X_1 B^\mu_1 + ig' X_2 B^\mu_2$. Here $T^a, (a=1,2,3)$ are the generators of $SU(2)$. The gauge couplings can be parametrized as $g = e/\sin \theta$, $g' = e/\cos \theta \cos \phi$ and $g'_1 = e/\cos \theta \sin \phi$. Here $\theta$ is the weak mixing angle and $\phi$ is a new mixing angle. The value of $\sin^2 \phi$ should be smaller than 1/2 since this would mean interchanging the labeling of the $U(1)$’s. Furthermore, in TC2 one of the $U(1)$’s is strong, choosing it to be $U(1)_1$, one has $\alpha_2 = g'_2/4\pi = O(1)$ and this gives roughly $\sin^2 \phi = O(0.1)$. We now rotate the $B^\mu_1$, $B^\mu_2$ fields in terms of $\phi$ as follows:

$$B^\mu = \cos \phi B^\mu_1 + \sin \phi B^\mu_2, \quad (1a)$$

$$Z^\mu_2 = \cos \phi B^\mu_1 - \sin \phi B^\mu_2. \quad (1b)$$

This choice of basis guarantees that $B^\mu$, coupling to $Y$, remains massless after the first stage of the breaking pattern. The new gauge boson, $Z_2$, gets most of its mass at this stage.

For the second stage of symmetry breaking, we rotate the $B^\mu$ and $W^\mu_3$ fields in terms of $\theta$, as in the standard model:

$$A^\mu = \cos \theta B^\mu + \sin \theta W^\mu_3, \quad (2a)$$

$$Z^\mu_3 = \cos \theta W^\mu_3 - \sin \theta B^\mu. \quad (2b)$$

Here we adopt a Higgs-like formalism for the sake of future clarity in notation. In TC2 fermions do not acquire masses via fundamental scalars. If one insists on using fundamental scalars to generate fermion masses one will need more than two doublets to provide generational mixing via neutral currents of the new gauge boson since models with less than three Higgs doublets are equipped with a natural Glashow-Iliopoulos-Maiani (GIM) mechanism.
The use of $SU(2)$-doublet condensates at this stage provides mass to $Z_1$. The heavier boson, $Z_2$, also gets some additional mass and $A$ remains massless. The currents to which these gauge bosons couple, are as follows:

\begin{align}
A \text{ to } Q=\bar{T}^3+Y \text{ with strength } e, \quad (3a) \\
Z_1 \text{ to } C=\bar{T}^3-Q \sin^2 \theta \text{ with } g_Z=e/\cos \theta \sin \theta, \quad (3b) \\
Z_2 \text{ to } C'=Y_1-Y \sin^2 \phi \text{ with } g_{Z'}=e/\cos \theta \sin \phi \cos \phi. \quad (3c)
\end{align}

Generally, the mass matrix for these gauge bosons is not diagonal, because the $SU(2)$-doublet condensates couple to both $Z_1$ and $Z_2$. Following the formalism in [2] we write the mass eigenstates as follows:

\begin{align}
Z &= Z_1 - \frac{\tan \phi \sin \theta}{\eta} \left( 1 + \frac{\xi}{\sin^2 \phi} \right) Z_2, \quad (4a) \\
Z' &= \frac{\tan \phi \sin \theta}{\eta} \left( 1 + \frac{\xi}{\sin^2 \phi} \right) Z_1 + Z_2. \quad (4b)
\end{align}

Here we introduced

\begin{align}
\xi &= \sum_j \langle T^3 Y_j \rangle_j / \sum_j \langle T^3 T^3 \rangle_j, \quad (5a) \\
\eta &= \frac{\sin^2 \theta}{\sin^2 \phi \cos^2 \phi} \frac{\sum_j \langle C' C' \rangle_j}{\sum_j \langle T^3 T^3 \rangle_j}. \quad (5b)
\end{align}

We make use of Higgs language for the sake of notation; $j$ runs over the condensates used to trigger the breaking stages and $\langle X \rangle_j$ means the vacuum expectation value (VEV) of $X$ with respect to the $j$th condensate. Now, in natural TC2 models the technifermion $Y_2$ hypercharges can be taken to be isospin symmetric [5]. Then, since $Q=\bar{T}_3+Y$ is conserved, the only contribution to $\xi$ comes from the top-quark condensate. This is

\begin{align}
\xi &= 2 f_t^2 (Y_{1L}-Y_{1R}), \quad (6)
\end{align}

with $v \approx 250$ GeV and $f_t \approx 64$ GeV [2], the top-pion decay constant. Thus, $\xi \\approx 0.13 (Y_{1L}-Y_{1R})$. The masses of the eigenstates in Eq. (4) are

\begin{align}
M_Z^2 &= M_{ZSM}^2 \left( 1 - \frac{\tan^2 \phi \sin^2 \theta}{\eta} \left( 1 + \frac{\xi}{\sin^2 \phi} \right)^2 \right), \quad (7a) \\
M_{Z'}^2 &= \eta M_{ZSM}^2 \left( 1 + \frac{\tan^2 \phi \sin^2 \theta}{\eta} \left( 1 + \frac{\xi}{\sin^2 \phi} \right)^2 \right). \quad (7b)
\end{align}

Here $M_{ZSM}$ is the standard model prediction for the mass of $Z$. Then the correction to $\rho$ parameter due to the shift in the $Z$ mass is given by

\begin{align}
\delta \rho_{Z'} &= \frac{\tan \phi \sin \theta}{\eta} \left( 1 + \frac{\xi}{\sin^2 \phi} \right)^2. \quad (8)
\end{align}

If $\xi = - \sin^2 \phi$, there will be no $Z_1-Z_2$ mixing, but the generation mixing effects of $Z_2$ will remain. The shift in $\rho$ must not be bigger than a percent [6]. So, to this order, we must have

\begin{align}
\eta &= \frac{M_{Z'}}{M_Z}. \quad (9)
\end{align}

In TC2, we expect $M_{Z'} \approx 1-2$ TeV, meaning that $\eta \approx 100$. We now rewrite the currents $Z(Z')$ couple with $g_Z(g_{Z'})$ factored out:

\begin{align}
J_Z &= C - \xi \sqrt{\delta \rho} \; C', \quad (10a) \\
J_{Z'} &= C' + \frac{g_{Z'}}{g_Z} \xi \sqrt{\delta \rho} \; C. \quad (10b)
\end{align}

Here, we suppressed chirality subscripts on $C$ and $C'$ for the sake of notation and omitted the $Z'$ subscript from $\delta \rho_{Z'}$. We also used $\xi = (g_Z M_Z)/(g_{Z'} M_{Z'}) \approx O(0.1)$ and $\delta \rho$ to have a compact and model independent notation.

If we consider $C$ and $C'$ as matrices in the 3-dimensional generation space, $C$ is a multiple of the identity, since the standard model $Z$ has universal couplings. Thus, $C$ will commute with the rotation matrices that bring the leptons to their mass eigenbasis. However, there is no a priori reason for $C'$ to be a multiple of identity and in TC2 it is not; after rotating the fermion fields, a nonuniversal $C'$ will induce tree-level generation mixing. We denote the rotated mass-eigenstate-charged lepton fields as follows (chiral indices are suppressed):

\begin{align}
\psi' = \Lambda \psi'. \quad (11)
\end{align}

Here $\psi'$, $(l=e, \mu, \tau)$, are the lepton mass eigenstates and $\Lambda$ is a $3 \times 3$ unitary matrix. As mentioned above $C'$ will not commute with the rotation matrices, so we introduce the rotated lepton vertex matrices

\begin{align}
L = \bar{\Lambda} C' \Lambda. \quad (12)
\end{align}

One of the biggest problems of such a theory is the flavor-changing neutral currents involving the first two generations. This is cured in TC2 by having the new gauge boson $Z_2$ couple with equal strength to the first two generations, and differently to the third. This implies the following:

\begin{itemize}
\item Clearly, our motivation is independent of the fact that whether neutrinos have mass or not, because the tree-level mixing is due to a neutral gauge boson.
\end{itemize}
FIG. 1. The 95% confidence level lower bound on $M_{Z'}$ resulting from the fit of the TC2 model we are considering to the precision $Z$ data. We reproduced the graph from Chivukula and Terning [2].

$$L^e\mu = (C' - C')\bar{X}^e\gamma^\mu,$$

(13a)

$$L^\tau = (C' - C')\bar{X}^\tau\gamma^\tau, \; l = e, \mu.$$  

(13b)

Then, if one assumes $\Lambda^\tau \gg \Lambda^\mu$, $l = e, \mu$, one has

$$L^\mu, L^\tau \gg L^\mu.$$  

(14)

Thus mixing between the first two generations is suppressed from the outset.

In what follows, we will present the results for $\mu \rightarrow e\gamma$, the electron’s electric dipole moment, $\mu \rightarrow 3e$ and $\mu - e$ conversion in Ti. To get a feel for the numerical implications, we will use a TC2 model for which $Y_1 = 0$ ($C' = Y \sin^2 \phi$) for the first two generations and $Y_1 = Y [C' = Y (1 - \sin^2 \phi)]$ for the third one. This results in $\xi \approx -0.07$. Chivukula and Terning [2] fit the full precision $Z$ data including atomic parity violations to this model. The results of their fit are summarized in Fig. 1. As can be seen from this graph, the lower bound for $M_{Z'}$ is around 1–2 TeV.

III. LEPTON MIXING PROCESSES

A. The amplitude for $l \rightarrow l' \gamma$

We calculated the amplitude for $l \rightarrow l' \gamma$ in a generalized $R_\xi$ gauge following the formalism of Ref. [7]. This process occurs at one loop; the photon couples to the internal lepton propagator and the loop closes with a $Z$ or $Z'$ line. The matrix element is $\mathcal{M} = e_{L}^\mu(q)\bar{u}(p')\Gamma^\mu u(p)$ and $\Gamma^\mu$ is given by

$$\Gamma^\mu = -i\frac{eG_Z^2}{16M_{Z'}^2\pi^2}[F^{\mu\nu}q_{\nu} - F^{\mu'\nu'}q_{\nu'}\gamma^5].$$  

(15)

with

$$F_{\pm} = \left[\frac{1}{3}[m_{L,R}]_{\pm} - \frac{2\sqrt{\delta \rho}}{\xi} C_{L}[m_{L,R}]_{\pm} - L_{\pm}m_{R}\right] \pm (l \rightarrow R).$$  

(16)

Here $[x,y]_\pm = xy \pm yx$ and $m$ is the mass matrix of the leptons. From this amplitude one can calculate the decay $l \rightarrow l' \gamma$ and the electric dipole moments for $l$.

1. The electron electric dipole moment

The electric dipole moments are given by the coefficient of $\sigma^{\mu\nu}q_{\nu}\gamma^5$ in Eq. (15), so for the electron we need to evaluate $F^{\mu\nu}$. This gives

$$d^e = \frac{eG_Z^2}{16M_{Z'}^2\pi^2}\text{Im}(m_{L,R}^{\mu\nu} + m_{\mu}L_{\mu}^{\mu\nu}).$$  

(17)

Assuming Eq. (14) holds, the RHS of the equation above is dominated by the first term. The experimental value for $d^e$ is $(-0.27 \pm 0.83) \times 10^{-26} \text{e cm}$ [6]. Taking the $Z'$ contribution to lie within $1\sigma$, we get the following constraint:

$$M_{Z'} \gtrsim \frac{3.93}{\sin \phi \cos \phi} \text{Im}(\bar{X}_{L}^{\mu\nu}\Lambda_{L}^{\mu\nu}\Lambda_{R}^{\mu\nu})^{1/2} \text{ TeV}.$$  

(18)

If one considers the magnitude of the quantity above ignoring the phases and assuming $\Lambda_{L} = \Lambda_{R} = K$ one finds $M_{Z'} \gtrsim \frac{0.14}{(\sin \phi \cos \phi)} \text{ TeV}$. Had we used $\Lambda_{L} = \Lambda_{R} = \sqrt{K}$ this would change to $M_{Z'} \gtrsim \frac{0.02}{(\sin \phi \cos \phi)} \text{ TeV}$. Recalling that in TC2 one expects $M_{Z'}$ to be around 1–2 TeV, the former gives $\sin^2 \phi \approx 0.01$ and the latter $\sin^2 \phi \approx 4 \times 10^{-4}$. These are expected, because very small values of $\sin^2 \phi$ would make $g_{Z'}$ diverge and this in turn will result in a large mass for $Z'$. On the other hand one can get rid of the electric dipole moments by assuming $\Lambda_{R} = \Lambda_{R} = 0, \; i \neq \tau$, leaving $\Lambda_{\tau}$ unconstrained (or vice versa). In the context of TC2, this type of behavior was strongly advocated for quark mixing angles to naturally eliminate the very stringent constraints resulting from $B_{u}^0 - \bar{B}_{d}^0$ mixing [8].

2. $\mu \rightarrow e \gamma$

Using the amplitude in Eq. (15) we find that the decay rate is

$$\Gamma(\mu \rightarrow e\gamma) = \alpha e\frac{G_Z^2 m_{\mu}}{2048 \pi^4 M_{Z'}^4} \left[|F^{\mu\nu}|^2 + |F^{\mu\nu'}|^2\right].$$  

(19)

With BR($\mu \rightarrow e\gamma$)$< 4.9 \times 10^{-11}$ [6] and assuming for simplicity $\Lambda_{R} = \Lambda_{R} = 0, \; i \neq \tau$, we have the following:

$$\xi^2 (C_{L}'^{\mu})^2 - (C_{L}'^{\nu'})^2 \left| 1 - 1.5\frac{\sqrt{\delta \rho}}{\xi (C_{L}'^{\mu} + C_{L}'^{\nu'})} \right| \lesssim 7.2 \times 10^{-4} \frac{\Lambda_{\mu}^{\mu\nu} \Lambda_{L}^{\mu\nu}}{\Lambda_{L}^{\mu\nu}}.$$  

(20)
Had we not used $\Lambda_Q^T = \Lambda_R^T = 0$, $i \neq \tau$ the amplitude would be dominated by $L_L m L_R$, which would make the RHS of Eq. (20) smaller by an amount $m_q/m_z = 0.06$. Even this will not help fix $Z'$ parameters; as we shall see shortly other lepton number violating modes are better by orders of magnitude.

**B. $\mu \rightarrow 3e$**

This decay mode is allowed at tree level. One finds for the decay rate

$$\Gamma(\mu \rightarrow 3e) = \frac{m_\mu^5}{768 \pi^3} \left( \frac{g_{Z'^2}}{2M_{Z'}} \right)^4 (3X + X').$$

With $X$ and $X'$ given by

$$X = \left| L_{\nu}^\mu \right|^2 + \left| L_{\nu}^\mu \right|^2 \left( B_{\nu}^{ee} \right)^2 + \left( B_{\nu}^{ee} \right)^2, \quad (22a)$$

$$X' = 2 \left| L_{\nu}^\mu \left( L_{\nu}^\mu \right)^* + \left( L_{\nu}^\mu \right)^* \left( L_{\nu}^\mu \right)^* \right| B_{\nu}^{ee} B_{\nu}^{ee}. \quad (22b)$$

Here, we defined $B_{\nu}^{ee} = L_{\nu}^{ee} - \left( \frac{\sqrt{\rho}}{C} \right) C_{\nu}^{ee} = (B_{\nu}^{ee})^*$. Using BR($\mu \rightarrow 3e) < 10^{-12}$ [6] and assuming, for simplicity, $\Lambda_R^T = \Lambda_R^T = 0$, $i \neq \tau$, we have

$$\frac{\xi^2 |C_L^{ee} (C_L^{ee} - C_L^{ee})|^2}{1 - 0.03 \frac{\sqrt{\rho}}{\xi C_L^{ee}} + 0.03 \frac{\rho}{\xi^2 (C_L^{ee})^2}} \leq 2.2 \times 10^{-7} \left| \Lambda_{LL}^T \Lambda_{LL}^{*T} \right|. \quad (23)$$

This is by far a better constraint than the one imposed by $\mu \rightarrow e \gamma$. The polynomial under the square root in Eq. (23) is stable between 1.00–1.06 for $\sin^2 \phi$ between 0.04–1. Thus, the constraint is dominated by the term multiplying the square root. We compare this process with the $\mu - e$ conversion in Ti in the following subsection.

**C. $\mu - e$ conversion in Ti**

This process is also a tree level process. Borrowing the result from Barneb"e et al. [9], one has the following rate (normalized to the muon capture rate $\Gamma_e$):

$$\mathcal{R}(\mu - e) = \frac{\alpha^3 m_e^2 Z^4 \xi^4 f_{ee} \left| L_{\nu}^{ee} \right|^2 \left| L_{\nu}^{ee} \right|^2}{3 \pi^2 Z \Gamma e M_{Z'}^2} X^2, \quad (24a)$$

$$X = (2Z + N) B_{\nu}^{uu} + (Z + 2N) B_{\nu}^{dd}, \quad (24b)$$

$$B_{\nu}^{uu} = C_{\nu}^{uu} - \frac{\sqrt{\rho}}{6} C_{\nu}^{qq}, \quad q = u, d. \quad (24c)$$

The parameters for Ti are, $Z = 22$, $N = 26$, $Z_{\text{eff}} = 17.6$, $\Gamma_e$ = $1.7 \times 10^{-15}$ MeV and $f = 0.54$ [9]. The current experimental upper bound $\mathcal{R}(\mu - e)_{\text{Ti}} < 4.3 \times 10^{-12}$ [6] will give (again assuming for simplicity $\Lambda_R^T = \Lambda_R^T = 0$, $i \neq \tau$)

$$\left| \xi^2 C_L^{ee} (C_L^{ee} - C_L^{ee}) \right| ^2 \leq 3.6 \times 10^{-8} \left| \Lambda_{LL}^T \Lambda_{LL}^{*T} \right|. \quad (25)$$

The term $\left| 1 + 0.27 \sqrt{\rho} / (\xi C_L^{ee}) \right|$ is stable between 0.8–1 for $\sin^2 \phi$ between 0.04–1, thus the constraint is dominated by $\xi^2 |C_L^{ee} (C_L^{ee} - C_L^{ee})|^2$ as in the case of $\mu \rightarrow 3e$. So we see that RHS of Eq. (25) is roughly 6 times the RHS of Eq. (23). This observation will remain roughly valid with the relaxation of the assumption $\Lambda_R^T = \Lambda_R^T = 0$, $i \neq \tau$. So we can disregard the process $\mu \rightarrow 3e$ and concentrate only on $\mu - e$ conversion for the numerical analysis. Since the rate for $\mu - e$ conversion depends on $\left| L_{\nu}^{ee} \right|^2 + \left| L_{\nu}^{ee} \right|^2$ the correct form of the constraint equation is

$$\xi^2 \left| C_L^{ee} (C_L^{ee} - C_L^{ee}) \right| \left| 1 + 0.14 \sqrt{\rho} / \xi C_L^{ee} \right| \leq 3.6 \times 10^{-8} / \delta. \quad (26)$$

Here

$$\delta = \sqrt{\left| \Lambda_{LL}^T \Lambda_{LL}^{*T} \right|^2 + 4 \left| \Lambda_{LL}^T \Lambda_{LL}^{*T} \right|^2}. \quad (27)$$

The factor of 4 in Eq. (27) is the ratio $(Y_R/Y_L)^2$ for leptons.

**IV. NUMERICAL ANALYSIS AND CONCLUSION**

In terms of the relevant quantities the constraint equation (26) reads

$$\left( \frac{\text{TeV}}{M_{Z'}} \right)^2 \frac{1}{s^2 c^2} \left| C_L^{ee} (C_L^{ee} - C_L^{ee}) \right| \left| 1 + 3.2 \sqrt{\rho} / \xi C_L^{ee} \right| \left( \frac{M_{Z'}}{\text{TeV}} \right)^{3/2} \leq 1.9 \times 10^{-5} / \delta, \quad (28)$$

where $s = \sin \phi$ and $c = \cos \phi$. Taking the lower limits for $M_{Z'}$ from Fig. 1 and feeding them in the constraint equation for $\mu - e$ conversion, Eq. (28), we get the upper limits on $\delta$ presented in Fig. 2. As can be seen from Fig. 2 the lowest upper bound for $\delta$ occurs at $\delta = 0$, and it increases steeply for smaller values of $\sin^2 \phi$. This is because $g_{Z'}$ diverges for...
vanishingly small values of $\sin^2 \phi$. The numerical upper bound for $\delta$ lies between $10^{-4}$ and $10^{-6}$ for the particular scenario we have considered. These values are compatible with the naive estimates made by taking $\Lambda \approx K$ or $\Lambda \approx \sqrt{K}$ for the mixing matrices. The constraint equation (28) is also sensitive to the $Z'$ hypercharges: for example in the TC2 model proposed recently by Lane [10] the bound is more stringent by a factor of 10.

Since, within TC2, the reasons for expecting the $Z'$ mass around $1 - 2$ TeV are somewhat robust, more stringent constraints may rule out lepton mixing altogether. For example, if the MECO experiment reaches the proposed precision of $10^{-16}$ for $\mu - e$ conversion in Ti without observing any candidate event, the upper bound on $\delta$ will be smaller by a factor of $5 \times 10^{-3}$. This will be hard to accommodate with reasonable $Z'$ mass and hypercharges and will lead to the exclusion of lepton number violation via $Z'$ for the TC2 model we have considered.

The main conclusion to be drawn is that the possibility of lepton number violation in TC2 remains an interesting feature for now. The proposed MECO experiment could tell which one of the present models involving lepton number violation may survive.

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