Phys. 402
Spring 2010
Qu-Ex 1 – Solutions

1) $t_R = t - \frac{R(t_R)}{c}$

$$\frac{\partial t_R}{\partial t} = 1 - \frac{1}{c}\frac{\partial t_R}{\partial t}\frac{\partial R}{\partial t_R}$$

$$\frac{\partial t_R}{\partial t} = \frac{1}{1 + \frac{1}{c}\frac{\partial R}{\partial t_R}}$$

$$\frac{\partial}{\partial t_R}\underbrace{\sqrt{r^2 - 2\vec{r}\cdot\vec{r}_0(t_R) + r_0^2(t_R)}}_{R} = -\frac{2\vec{r}\cdot\vec{v}(t_R) + 2\vec{r}_0\cdot\vec{v}}{2\sqrt{\quad}}$$

$$\frac{\partial R}{\partial t_R} = -\hat{n}\cdot\vec{v}$$

$$\boxed{\frac{\partial t_R}{\partial t} = \frac{1}{1 - \hat{n}\cdot\vec{\beta}}}$$

2) $$\frac{\partial R}{c\,\partial t} = \frac{\partial t_R}{\partial t}\frac{1}{c}\frac{\partial R}{\partial t_R} = \frac{1}{(1-\hat{n}\cdot\vec{\beta})}\left(-\hat{n}\cdot\frac{\vec{v}}{c}\right) = -\frac{\hat{n}\cdot\vec{\beta}}{1-\hat{n}\cdot\vec{\beta}}$$

$$\boxed{\frac{1}{c}\frac{\partial R}{\partial t} = -\frac{\hat{n}\cdot\vec{\beta}}{1-\hat{n}\cdot\vec{\beta}}}$$

3) $\nabla ct_R = \nabla (ct - R) = -\nabla R$

$R = R(\vec{r}, t_R)$ and $t_R = t_R(t, \vec{r})$

$$\frac{\partial}{\partial \vec{r}} R = \frac{\partial}{\partial \vec{r}} R\Big|_{t_R} + \frac{\partial t_R}{\partial \vec{r}} \cdot \frac{\partial \vec{R}}{\partial t_R}\Big|_{\vec{r}}$$

(t_R: kept constant; $\vec{r}$: kept constant)

$$\nabla R\Big|_{t_R} = + \frac{2(\vec{r} - \vec{r}_0)}{2|\vec{r} - \vec{r}_0|} = +\hat{n}$$

$$\frac{\partial \vec{R}}{\partial t_R}\Big|_{\vec{r}} = -\hat{n} \cdot \vec{v} \quad \text{(from (1))}$$

$$\nabla R = +\hat{n} + \nabla t_R (-\hat{n} \cdot \vec{v})$$

so $\nabla ct_R = c \nabla t_R = -\frac{\hat{n}}{c} + \nabla t_R \, \hat{n} \cdot \frac{\vec{v}}{c}$

$$\nabla t_R = \frac{-\hat{n}/c}{1 - \vec{\beta} \cdot \hat{n}}$$

$$\boxed{\nabla ct_R = -\frac{\hat{n}}{1 - \vec{\beta} \cdot \hat{n}}}$$

4) $\nabla R = \hat{n} + \left(-\frac{\hat{n}/c}{1 - \vec{\beta} \cdot \hat{n}}\right)(-\hat{n} \cdot \vec{v}) = \frac{\hat{n}}{1 - \hat{n} \cdot \vec{\beta}}$

$$\boxed{\nabla R = -\nabla ct_R = \frac{\hat{n}}{1 - \hat{n} \cdot \vec{\beta}}}$$