Phys. 402

Spring 2010

Qu-Ex 2 - Solutions

a) $Q_p(t) = \int d\bar{r}'\, \rho(\bar{r}',t)\,[3\bar{r}'\bar{r}' - \mathbb{1} r'^2]$

$$\rho(\bar{r}',t) = q\, \delta(x')\,\delta(y')\,\delta(z' - D\cos\omega_0 t)$$

$$Q_{phys}(t) = qD^2\cos^2\omega_0 t\,(3\hat{z}\hat{z} - \mathbb{1}) = qD^2\cos^2\omega_0 t\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&2\end{pmatrix}$$

$$\cos^2\omega_0 t = \frac{1+\cos 2\omega_0 t}{2} = \frac{1}{2}\mathrm{Re}\,(1+e^{-2i\omega_0 t})$$

$\omega = 2\omega_0$

$$\underline{\underline{Q_0}} = \begin{pmatrix}-1&0&0\\0&-1&0\\0&0&2\end{pmatrix} \qquad Q_{phys}(t) = \frac{qD^2}{2}\underline{\underline{Q_0}}\,\mathrm{Re}\,(1+e^{-i(2\omega_0)t})$$

$\omega = 2\omega_0 \quad Q_0 = \begin{pmatrix}-1&0&0\\0&-1&0\\0&0&2\end{pmatrix}$

b) $\vec{Q}_0 = \underline{\underline{Q_0}}\cdot\hat{r} = (-n_x, -n_y, 2n_z)$

$|Q_{0\perp}|^2 = \vec{Q}_0\cdot\vec{Q}_0^* - |\hat{r}\cdot\vec{Q}_0|^2 = \qquad \hat{r}\cdot\vec{Q}_0 = -n_x^2 - n_y^2 + 2n_z^2$

$$= n_x^2 + n_y^2 + 4n_z^2 - (-n_x^2 - n_y^2 + 2n_z^2)^2$$

$$= \sin^2\theta + 4\cos^2\theta - (-\sin^2\theta + 2\cos^2\theta)^2$$

$$= \sin^2\theta + 4\cos^2\theta - (\sin^4\theta - 4\sin^2\theta\cos^2\theta + 4\cos^4\theta)$$

$$= \sin^2\theta(1-\sin^2\theta) + 4\cos^2\theta(1-\cos^2\theta) + 4\sin^2\theta\cos^2\theta = 9\sin^2\theta\cos^2\theta$$

$$\left.\frac{dP}{d\Omega}\right|_{E2} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{1}{288\pi}\cdot\frac{2^6\omega_0^6}{c^5}\;\frac{q^2D^4}{4}\cdot 9\sin^2\theta\cos^2\theta =$$

$$= \left(\frac{1}{4\pi\epsilon_0}\right)\frac{\omega_0^6 q^2 D^4}{2\pi}\sin^2\theta\cos^2\theta$$

c) Because there is no dipole radiation at frequency $\omega = 2\omega_0$.

$\vec{p}_{phys}(t) = qD\cos\omega_0 t\,\underline{\hat{z}} = qD\,\mathrm{Re}\,e^{-i\omega_0 t}\,\underline{\hat{z}}$ radiates at $\omega = \omega_0$.