Phys. 402

Spring 2010

QuEx. 3 – Solutions

a)

$$J_{Med} = \begin{pmatrix} c\rho_P \\ \vec{j}_{pol} + \vec{j}_{Mag} \end{pmatrix} = \begin{pmatrix} -c\nabla\cdot\vec{P} \\ \frac{\partial \vec{P}}{\partial t} + \nabla\times\vec{M} \end{pmatrix}$$

b) Free case

$$c\rho_F = c\nabla\cdot\vec{E}\,\epsilon_0$$

$$\vec{J}_F = \epsilon_0\left(c^2\nabla\times\vec{B} - \frac{\partial\vec{E}}{\partial t}\right)$$

$$\epsilon_0\vec{E} \rightarrow -\vec{P}$$

$$\vec{E} \rightarrow -\vec{P}/\epsilon_0$$

$$\epsilon_0 c^2\vec{B} \rightarrow M$$

$$\vec{B} \rightarrow \frac{\vec{M}}{\epsilon_0 c^2}$$

so, $\Sigma = F\begin{pmatrix} \vec{E} \rightarrow -\vec{P}/\epsilon_0 \\ \vec{B} \rightarrow \vec{M}/\epsilon_0 c^2 \end{pmatrix}$

$$\boxed{\Sigma = \frac{1}{\epsilon_0}\begin{pmatrix} 0 & \vec{P}c \\ -\vec{P}c & -\underline{\underline{\epsilon}}\cdot\vec{M} \end{pmatrix}}$$

c) We use the $\vec{E}\,\vec{B} \to \vec{E}'\vec{B}'$ transformation formulas given in the hint, and make the replacement $\vec{E} \to -\vec{P}/\epsilon_0$, $\vec{B} \to \vec{M}/\epsilon_0 c^2$

and find

$$\vec{P}'_{/\!/} = \vec{P}_{/\!/} \qquad \vec{M}'_{/\!/} = \vec{M}_{/\!/}$$

$$\vec{P}'_{\perp} = \gamma\left(\vec{P}_{\perp} - \frac{\vec{u}\times\vec{M}}{c^2}\right) \qquad \vec{M}'_{\perp} = \gamma\left(\vec{M}_{\perp} + \vec{u}\times\vec{P}\right)$$