Physics 402

Spring 2010

QuEx_3

a) Bring the ME (with matter) to the form:

$$\partial_a F^{ab} = \frac{j_F^b}{\epsilon_0} + \frac{j_{medium}^b}{\epsilon_0} .$$

What are the time and space components of the 4-vector j_{Medium}?

b) Find a two-tensor Σ^{ab} such that

$$\partial_a \Sigma^{ab} = J^b_{medium} / \epsilon_0$$

(Hint: Use the definition of J_{medium} and the free-case ME that lead to $\partial_a F^{ab} = J_F^b/\epsilon_0$. Form a correspondence (analogy) with fields in Σ).

c) Find how $\vec{P}$ and $\vec{M}$ transform under a relativistic transformation (S) → (S')

(Hint: Consider the transformation of $\vec{E}$ and $\vec{B}$ fields in F^{ab}. Form a correspondence (analogy) with fields in Σ.)

Hints: (matter absent)

$$\partial_a F^{ab} = J_F^b/\epsilon_0$$

$$F^{ab} = \begin{pmatrix} 0 & -c\vec{E} \\ c\vec{E} & -\underset{\equiv}{\epsilon}\cdot(c^2\vec{B}) \end{pmatrix}, \quad J_F = \begin{pmatrix} c\rho_F \\ \vec{J}_F \end{pmatrix}$$

As $(S) \to (S')$ $\vec{E}, \vec{B}$ transform as:

$$\vec{E}'_{/\!/} = \vec{E}_{/\!/} \qquad \vec{B}'_{/\!/} = \vec{B}_{/\!/}$$

$$\vec{E}'_{\perp} = \gamma(\vec{E}_{\perp} + \bar{u}\times\vec{B}) \qquad \vec{B}'_{\perp} = \gamma\left(\vec{B}_{\perp} - \frac{\bar{u}\times\vec{E}}{c^2}\right)$$