Phys. 402

Spring 2010

QuEx4 _ Solutions

i) Tangential components of $\vec{E}$ and $\vec{H}$ are continuous across the boundary.

$E_{0i} + E_{0R} = E_{0T}$ $\qquad H_{0i} + H_{0R} = H_{0T}$

Faraday's law gives $\vec{H} = \frac{\vec{k} \times \vec{E}}{\omega \mu} = \hat{e}_z \frac{Ek}{\mu_0 \omega}$

$E_{0i} - E_{0R} = \frac{n_2}{n_1} E_{0T}$ $\qquad E_{0i} k_1 - E_{0R} k_1 = E_{0T} k_T$, $\frac{k_T}{k_1} = \frac{n_2}{n_1}$

This gives $\frac{E_{0T}}{E_{0i}} = \frac{2n_1}{n_1+n_2} = T$, $\frac{E_{0R}}{E_{0i}} = \frac{n_1 - n_2}{n_1+n_2} = R$

$\vec{E}_R = R E_{0i} \hat{x}\, e^{i(-k_1 z - \omega t)}$ $\qquad \vec{E}_T = T E_{0i} \hat{x}\, e^{i(k_2 z - \omega t)}$

ii) $\bar{S} = \frac{1}{2} \epsilon |E|^2 \frac{c}{n}$ $\qquad \frac{1}{2} \epsilon_1 |E_{0i}|^2 \frac{c}{n_1} \overset{?}{=} \frac{1}{2} \epsilon_1 |E_{0R}|^2 \frac{c}{n_1} + \frac{1}{2} \epsilon_2 |E_{0T}|^2 \frac{c}{n_2}$

or $\underbrace{|R|^2}_{R} + \overbrace{|T|^2 \frac{\epsilon_2}{\epsilon_1} \cdot \frac{n_1}{n_2}}^{T} \overset{?}{=} 1$ $\qquad \left(\frac{n_2}{n_1}\right)^2 \left(\frac{n_1}{n_2}\right) = \frac{n_2}{n_1}$

$\Rightarrow \frac{(n_1-n_2)^2}{(n_1+n_2)^2} + \frac{4n_1^2}{(n_1+n_2)^2} \cdot \frac{n_2}{n_1} = 1.$

iii) $R = \frac{(1-1.5)^2}{(1+1.5)^2} = \frac{(1/2)^2}{(5/2)^2} = \frac{1}{25} = 0.04$ $\qquad$ 4% intensity is reflected back.