Phys. 402

Spring 2010

QuEx5 _ Solutions

1) a) $\vec{J}(\vec{r},t) = q v_0 \, \delta(\vec{r}-\vec{v}_0 t)\, \theta(-t) + q\vec{v}\, \delta(\vec{r}-\vec{v}t)\, \theta(t)$

$$\vec{J}\left(\frac{\omega}{c}\hat{\underline{r}},\omega\right) = \int_{-\infty}^{0} dt'\, q\vec{v}_0\, e^{i\left(\omega - \frac{\omega}{c}\hat{\underline{r}}\cdot\vec{v}_0\right)t'} + \int_{0}^{+\infty} dt'\, q\vec{v}\, e^{i\left(\omega - \frac{\omega}{c}\hat{r}\cdot\vec{v}\right)t'}$$

$$\int_0^{\infty} dt'\, e^{iAt'} e^{-\epsilon t'} = -\frac{1}{iA} \qquad \int_{-\infty}^{0} dt'\, e^{+iAt'} e^{-\epsilon t'} = \frac{1}{iA}$$

$$\vec{J}\left(\frac{\omega}{c}\hat{\underline{r}},\omega\right) = \frac{q\ \vec{v}_0}{i(\omega)\left(1-\hat{\underline{r}}\cdot\frac{\vec{v}_0}{c}\right)} - \frac{q\vec{v}}{i\omega\left(1-\hat{\underline{r}}\cdot\frac{\vec{v}}{c}\right)}$$

$$\simeq \frac{q\,(\vec{v}_0-\vec{v})}{i\omega}$$

b) $$\frac{\partial^2 E}{\partial\omega\,\partial\Omega} = \left(\frac{1}{4\pi\epsilon_0}\right) \frac{\cancel{\omega^2}}{4\pi^2 c^3} \frac{q^2}{\cancel{\omega^2}} \underbrace{|\hat{r}\times(\vec{v}_0-\vec{v})|^2}_{\underbrace{\hat{r}\cdot\hat{r}\,(\vec{v}_0-\vec{v})^2 - (\hat{r}\cdot(\vec{v}_0-\vec{v}))^2}_{(\vec{v}_0-\vec{v})^2\sin^2\theta}}$$

$$= \left(\frac{1}{4\pi\epsilon_0}\right) \frac{q^2}{4\pi^2 c^3} (\vec{v}_0-\vec{v})^2 \sin^2\theta$$

c) $$\int d\Omega\, (1-\mu^2) = 2\pi \int_{-1}^{+1} d\mu\, (1-\mu^2) = 2\pi\left(2-\frac{2}{3}\right) = \frac{8\pi}{3}$$

$$\frac{dE}{d\omega} = \left(\frac{1}{4\pi\epsilon_0}\right) \frac{2q^2}{3\pi c^3} (\vec{v}_0-\vec{v})^2$$

2) TEM waves: $E_{03}^{TEM}=0$, $B_{03}^{TEM}=0$ $\quad \vec{k}=(k_1,0,k_3)$

$\nabla\cdot\bar{E} \Rightarrow -k_1 E_{01}^{TEM} + ik_3 \overset{\to 0}{\cancel{E_{03}^{TEM}}} = 0$ $\quad k_1=0$ for TEM waves.

$\bar{E}(\bar{r}) = \hat{e}_1 E_{01} e^{ik_3 z}$. $\quad \hat{E}(\bar{r},t) = E(\bar{r})\, e^{-i\omega t}$

$\nabla\times\bar{E} = i\omega\bar{B} = \begin{vmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \\ \partial_x & \partial_y & \partial_z \\ E_{01} & 0 & 0 \end{vmatrix}$ $\quad -\nabla\times E - \frac{\partial \bar{B}}{\partial t} = 0$

$\hat{B} = B(\bar{r})e^{-i\omega t}$.

$= \hat{e}_2\, ik_3 E_{0} e^{ik_3 z}$

$\bar{B} = \hat{e}_2 \frac{k_3}{\omega} E_{01} e^{ik_3 z}$ $\quad \omega^2 = c^2(\overset{\to 0}{\cancel{k_1^2}} + k_3^2) = c^2 k_3^2$

no cut-off for TEM waves.

The system <u>supports</u> TEM waves.