Phys. 402

Spring 2010

Qu Ex 5

1) A charged particle is moving with constant non-relativistic velocity $\vec{v}_0$ for times $t<0^-$. At time $t=0$ it receives an impulse and changes its velocity to non-relativistic $\vec{v}$, which is constant for times $t>0^+$.

a) Compute the FT of the current density:

$$\vec{J}(\vec{k},\omega)=\int_{-\infty}^{+\infty}dt'\int d\vec{r}'\, e^{-i(\vec{k}\cdot\vec{r}'-\omega t')}\,\vec{j}(\vec{r}',t')$$

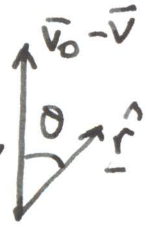
with $\vec{k}=\frac{\omega}{c}\hat{\underline{r}}$ and ignore $\beta_0=\frac{v_0}{c}$, $\beta_1=\frac{v_1}{c}$ with respect to 1.

(Hint: $\vec{j}(\vec{r},t)=\vec{J}_{before}\,\theta(-t)+\vec{J}_{after}\,\theta(t)$, $\theta(t)=\begin{cases}1 & t>0\\ 0 & t<0\end{cases}$

In order to make time integrals convergent, introduce a convergence factor $e^{-\epsilon|t|}$ $(\epsilon\to 0^+)$).

b) Compute the energy radiated per unit solid angle, per unit frequency:

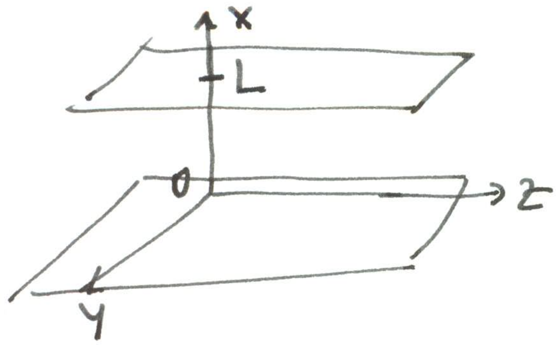
$$\frac{\partial^2 E}{\partial\omega\,\partial\Omega}=\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\omega^2}{4\pi^2c^3}\left|\hat{\underline{r}}\times\vec{J}(\vec{k},\omega)\right|^2,\quad \vec{k}=\frac{\omega}{c}\hat{\underline{r}},$$



(take $\vec{v}_0-\vec{v}$ as the z-axis, measure θ with respect to $\hat{z}$).

c) Compute the energy radiated per unit frequency

$$\frac{dE}{d\omega} = \int d\Omega \, \frac{\partial^2 E}{\partial\omega \, \partial\Omega}$$

2) 

Two infinite perfectly conducting plates are at $x=0$, and $x=L$ planes. Does this system support TEM waves in the region $0 \leq x \leq L$?

(Hint: with no loss of generality, assume TEM waves propagate only along the +z- direction).