Phys. 402
Spring 2010
Final Exam - Solutions

1) a) From Lecture notes

$$\vec{p}_{ed} = \underbrace{4\pi\epsilon_0 \left(\frac{K-1}{K+2}\right) a^3 E_0 \hat{z}}_{p_0} e^{-i\omega t} = p_0 \hat{z} e^{-i\omega t}$$

$$\frac{dP}{d\Omega} = \left(\frac{1}{4\pi\epsilon_0}\right) \frac{\omega^4}{8\pi c^3} |p_0|^2 \sin^2\theta$$

b) $$\langle \vec{S} \rangle = \frac{1}{\mu_0} \frac{1}{2} \mathrm{Re}(\vec{E}^* \times \vec{B}) \qquad \vec{B} = -\frac{E_0}{c} \hat{y} e^{i(kx-\omega t)}$$

$$= c\hat{z}\left(\epsilon_0 \frac{|E_0|^2}{2}\right) \qquad |\langle \vec{S} \rangle| = c\,\epsilon_0 \frac{|E_0|^2}{2}$$

$$\frac{d\sigma}{d\Omega} = \frac{\frac{1}{4\pi\epsilon_0} \frac{\omega^4}{8\pi c^3} |p_0|^2 \sin^2\theta}{c\,\epsilon_0 \frac{|E_0|^2}{2}} = \frac{\omega^4}{c^4}\left(\frac{K-1}{K+2} a^3\right)^2 \sin^2\theta$$

$$\dim\left(\frac{d\sigma}{d\Omega}\right) = L^2$$

c) $$\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{8\pi}{3}\left(\frac{K-1}{K+2} a^3\right)^2 \frac{\omega^4}{c^4}$$

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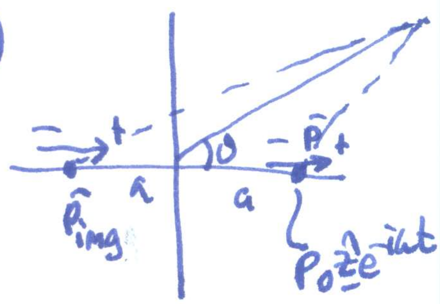
2)

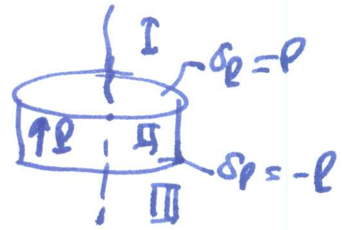


since $\lambda \gg a$ it is as if we have a dipole of $2\vec{p}$.

$$\frac{dP}{d\Omega} = \left(\frac{1}{4\pi\epsilon_0}\right) \frac{\omega^4}{8\pi c^3} \cdot 4|p_0|^2 \sin^2\theta$$

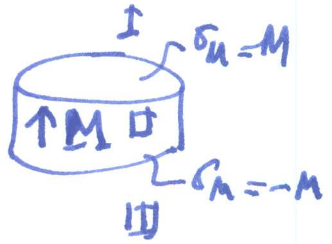
3)

a)



E_I	E_{II}	E_{III}	D_I	D_{II}	D_{III}
0	$-\frac{\rho}{\epsilon_0}$	0	0	0	0

b)



B_I	B_{II}	B_{III}	H_I	H_{II}	H_{III}
0	0	0	0	$-M$	0

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4) A)

i) $\vec{p}_T = \vec{p} - \vec{p} = 0$

ii) $\vec{m}$ constant $= 2\pi a^2 I \hat{z}$

iii) $\underline{\underline{Q}}$ radiates $\omega_r = 2\omega$

LM	ω_r
Q	2ω

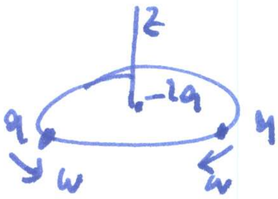
LM: lowest multipole radiating ω_r = frequency of radiation?

B)

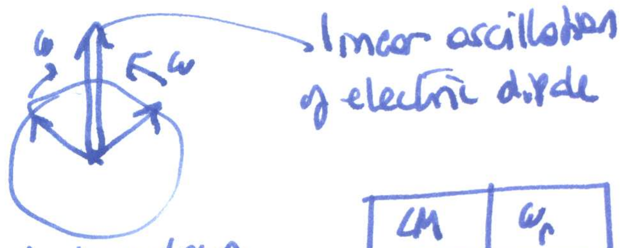
i) $\vec{p}$ dipole rad. ω

LM	ω_r
p	ω

c)



$\Longrightarrow$



Looking down from z-axis.

LM	ω_r
P	ω