HW2_1

# Phys 402
## Spring 2010
## Homework 2

1) show that

$$-\vec{a}_\perp + \hat{n} \times (\vec{a} \times \vec{\beta}) = \hat{n} \times \vec{K}$$

where $\vec{K} = (\hat{n} - \vec{\beta}) \times \vec{a}$

2) show that

$$\hat{n} \times \vec{K} \cdot \hat{n} \times \vec{K} = K^2 - (\hat{n} \cdot \vec{K})^2$$

$$= \left(1 - (\hat{n} \cdot \vec{\beta})\right)^2 a^2 + 2(1 - \hat{n} \cdot \vec{\beta})\, \vec{a} \cdot \vec{\beta}\ \ \hat{n} \cdot \vec{a}$$

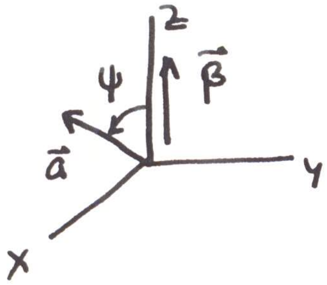
$$+ (\beta^2 - 1)(\vec{a} \cdot \hat{n})^2$$

so that

$$\frac{dP(t_r)}{d\Omega} = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q^2}{4\pi c^3}\right) \times \frac{1}{(1 - \hat{n} \cdot \vec{\beta})^5}$$

$$\times \Big( (1 - \hat{n} \cdot \vec{\beta})^2 a^2 + 2(1 - \hat{n} \cdot \vec{\beta})\, \vec{a} \cdot \vec{\beta}\ \ \hat{n} \cdot \vec{a}$$

$$+ (\beta^2 - 1)(\vec{a} \cdot \hat{n})^2 \Big)$$

3) 


$$P(t_n) = \int d\Omega \, \frac{dP(t_n)}{d\Omega}$$

i) Show that

$$P(t_n) = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q^2}{2c^3}\right)\int_{-1}^{+1} d\mu \left\{ \frac{a^2}{(1-\beta\mu)^3} + \frac{a^2(1-\beta^2)}{(1-\beta\mu)^5}\frac{(1-\mu^2)}{2} + (\vec{a}\cdot\vec{\beta})^2\left(\frac{2\mu}{\beta(1-\beta\mu)^4} - \frac{(1-\beta^2)}{\beta^2(1-\beta\mu)^5}\left(\frac{3\mu^2-1}{2}\right)\right)\right\}$$

ii) change variable $\omega = 1-\beta\mu$

$$\int_{-1}^{+1} d\mu \ldots \rightarrow \frac{1}{\beta}\int_{1-\beta}^{1+\beta} d\omega \ldots$$

set $I_n = \displaystyle\int_{1-\beta}^{1+\beta} \frac{d\omega}{\omega^n}$

We need, (show)

$$I_3 = \frac{2\beta}{(1-\beta^2)^2}, \quad I_4 = \frac{2\beta}{3}\frac{(3+\beta^2)}{(1-\beta^2)^3}, \quad I_5 = \frac{2\beta(1+\beta^2)}{(1-\beta^2)^4}$$

show that

$a^2$ terms: $\displaystyle \frac{1+\beta^2}{2\beta^2} I_3 - \frac{(1-\beta^2)}{\beta^2} I_4 + \frac{(1-\beta^2)^2}{2\beta^2} I_5$

$$= \frac{4\beta}{3(1-\beta^2)^2}$$

$(\vec{a}\cdot\vec{\beta})^2$ terms:

$$-\frac{(3+\beta^2)}{2\beta^4} I_3 + \frac{(3-\beta^2)}{\beta^4} I_4 - \frac{(1-\beta^2)(3-\beta^2)}{2\beta^4} I_5$$

$$= \frac{\frac{4}{3}\beta}{(1-\beta^2)^3}$$

4) i) Put everything together to obtain:

$$P(t_R) = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{2q^2}{3c^3}\right)$$

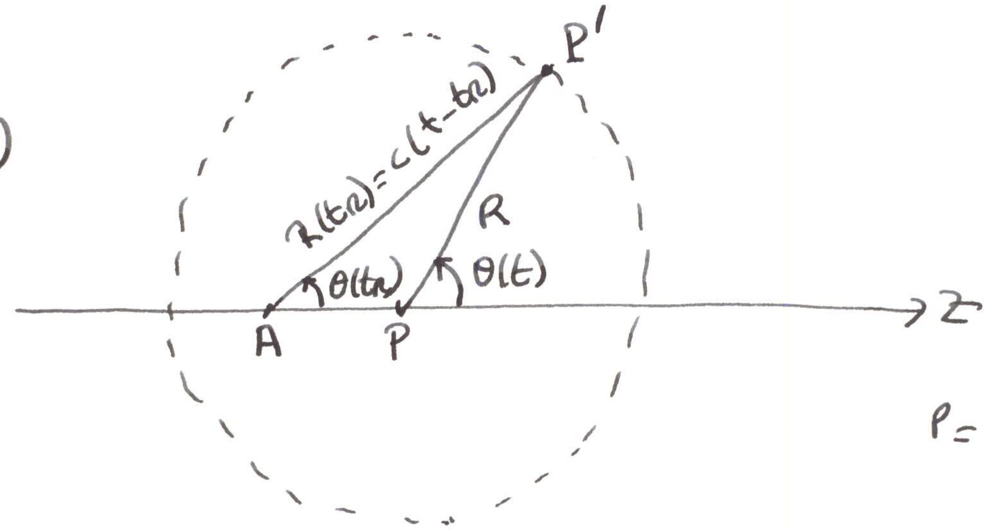
$$\times \frac{1}{(1-\beta^2)^2}\left(a^2 + \frac{(\vec{a}\cdot\vec{\beta})^2}{(1-\beta^2)}\right)$$

ii) Write $P(t_R)$ when

→ $\vec{a} \mathbin{//} \pm\vec{\beta}$

→ $\vec{a} \perp \vec{\beta}$

5)




P = charge at time t.

$R = |\bar{r} - \bar{r}_0(t)|$ $\quad\quad$ $AP = v(t - t_R)$ $\quad\quad$ $v$ = constant, $\bar{v} = v\hat{z}$

Problem is given P', find A (or $t_R$).

Show that

$$t_R = t - \frac{R}{c}\gamma^2\left(\beta\cos\theta(t) - \sqrt{1 - \beta^2\sin^2\theta(t)}\right)$$

(Hint: Consider the triangle APP'

$$(AP')^2 = (AP)^2 + (PP')^2 - 2(AP)(PP')\cos(\pi - \theta(t))\Big)$$

Once $t_R$ is found, $z_A = z_P - v(t - t_R)$.

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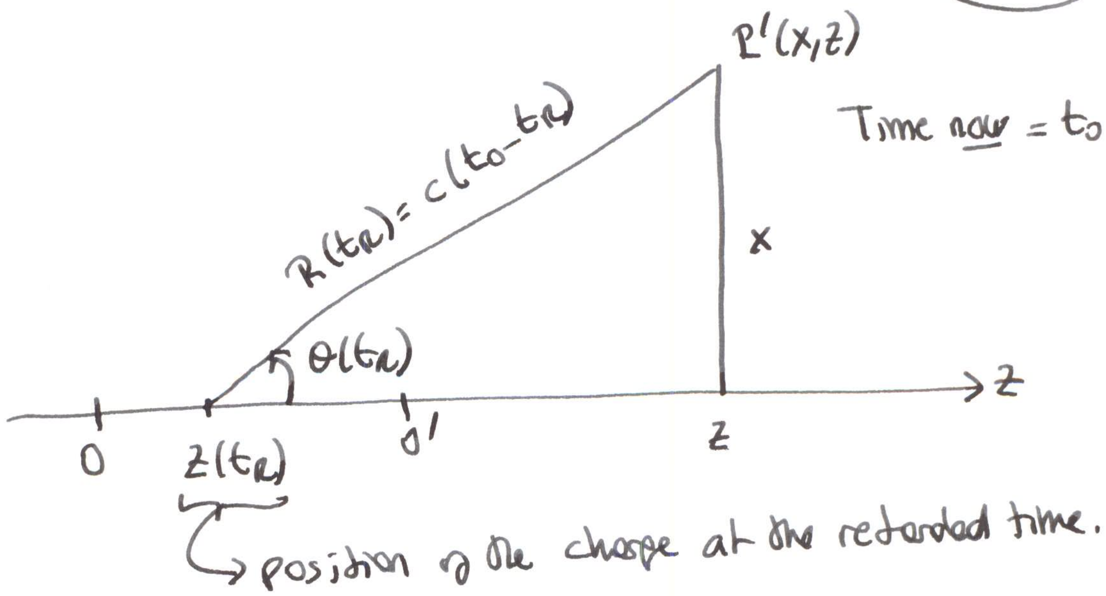
6)




Particle speed: for $z < z_0$ $\quad \bar{v} = \bar{v}_i \hat{z}$ constant.

for $z_0 \leq z \leq z_{0'}$ $\quad \bar{v} = \bar{v}_i + a t_R$

for $z \geq z_{0'}$ $\quad \bar{v}_F = \bar{v}_i + \bar{a} T \qquad a = \dfrac{\Delta v}{T}$

$$z(t_R) = v_i t_R + \frac{1}{2} a t_R^2 = v_i t_R + \frac{1}{2} \frac{\Delta v}{T} t_R^2$$

Problem: given $(x, z)$ (in the zone of the acceleration fields (see lecture notes).

find $t_R$. show that $t_R$ is the solution of the quartic equation

$$\left(\frac{t_R}{T}\right)^4 + a_3 \left(\frac{t_R}{T}\right)^3 + a_2 \left(\frac{t_R}{T}\right)^2 + a_1 \left(\frac{t_R}{T}\right) + a_0 = 0$$

where $\rightarrow$

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$$a_3 = \frac{4 v_i}{\Delta v}$$

$$a_2 = \frac{4 \left( v_i^2 - c^2 - z \, \Delta v / T \right)}{\Delta v^2}$$

$$a_1 = \frac{8 \left( c^2 t_0 - z v_i \right)}{\Delta v^2 \; T}$$

$$a_0 = \frac{4 \left( x^2 + z^2 - c^2 t_0^2 \right)}{\Delta v^2 \, T^2}$$

(Hint: $x^2 + (z - z(t_{rv}))^2 = c^2 (t_0 - t_{rv})^2$ )