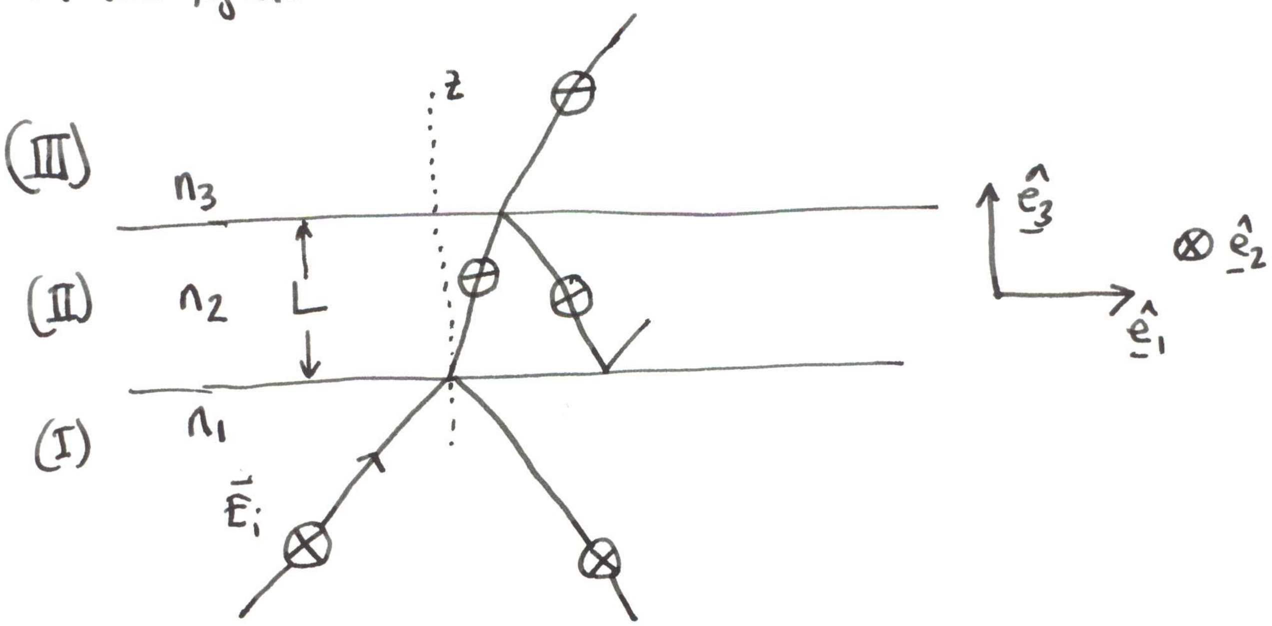
DIELECTRIC THIN FILM

and FRUSTRATED TOTAL INTERNAL REFLECTION (or, OPTICAL BARRIER PENETRATION)

A thin film of thickness L and index of refraction n_2 seperates two semi-infinite media with refractive indices n_1 and n_3 respectively (n_1, n_2, n_3 can be complex).

[A] TE (Transverse electric) plane waves are incident on the film with wave vector $\vec{k}_1 = k_{1x}\hat{e}_1 + k_{1z}\hat{e}_3$ as shown in the figure below



Let $\vec{E}_R$ be the reflected wave, $\vec{E}_T$ the transmitted wave, and $\vec{E}_+$ the resultant of all TE waves propagating in $(+\hat{e}_3)$ direction in the film, $\vec{E}_-$ the resultant of all TE waves propagating in $(-\hat{e}_3)$ direction in the film. Hence the TE fields in regions I, II, III are:

$$\vec{E}_I = E_{0i}\,\hat{e}_2\, e^{i(k_{1x}x + k_{1z}z - \omega t)} + E_{0R}\,\hat{e}_2\, e^{i(k_{1x}x - k_{1z}z - \omega t)}$$

$$\vec{E}_{II} = E_{0+}\,\hat{e}_2\, e^{i(k_{2x}x + k_{2z}z - \omega t)} + E_{0-}\,\hat{e}_2\, e^{i(k_{2x}x - k_{2z}z - \omega t)}$$

$$\vec{E}_{III} = E_{0T}\,\hat{e}_2\, e^{i(k_{3x}x + k_{3z}z - \omega t)}$$

(A1) Write the boundary conditions (4 equations)

(A2) Eliminate E_{0+} and E_{0-} from the boundary condition equations to show that $R_\perp^E$ and $T_\perp^E$ are related by a transfer matrix $M_\perp^E$

$$\begin{bmatrix} 1 \\ k_{1z} \end{bmatrix} + \begin{bmatrix} 1 \\ -k_{1z} \end{bmatrix} R_\perp^E = M_\perp^E \begin{bmatrix} 1 \\ k_{3z} \end{bmatrix} T_\perp^E \, e^{ik_{3z}L}$$

where

$$M_\perp^E = \begin{pmatrix} \cos k_{2z}L & -\frac{i}{k_{2z}} \sin k_{2z}L \\ -ik_{2z} \sin k_{2z}L & \cos k_{2z}L \end{pmatrix}$$

Show that $M_\perp^E$ is unimodular (i.e., $\det(M_\perp^E) = +1$) always (i.e., for real, imaginary or complex values of k_{2z}).

(A3) Solve the boundary condition equations for $R_\perp^E$ and obtain

$$R_\perp^E = \frac{k_{2z}(k_{1z} - k_{3z}) \cos k_{2z}L + i(k_{2z}^2 - k_{1z}k_{3z}) \sin k_{2z}L}{k_{2z}(k_{1z} + k_{3z}) \cos k_{2z}L - i(k_{2z}^2 + k_{1z}k_{3z}) \sin k_{2z}L}$$

What is $R_\perp^E$ for normal incidence ($\vec{k}_1 = k_1 \hat{e}_3$)

$R_\perp^E \rightarrow R_{Normal} = ?$

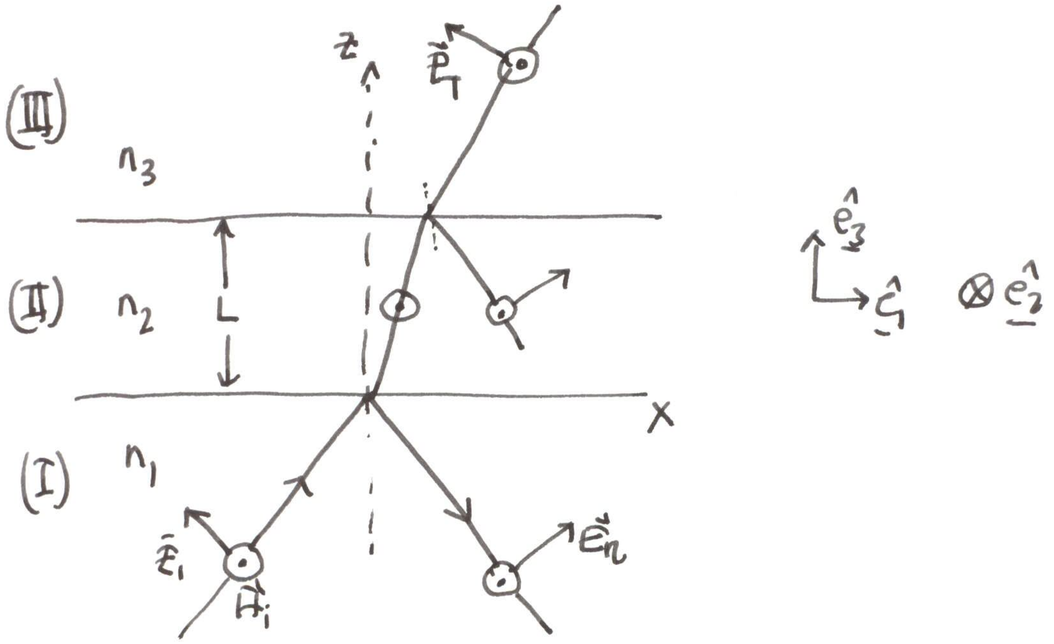
(A4) The result of the multiple reflection calculation gives for the reflection amplitude

$$R = \frac{r_{12} + r_{23}\, e^{2ik_{2z}L}}{1 + r_{12} r_{23}\, e^{2ik_{2z}L}}$$

What are r_{12} and r_{23} for the oblique incidence of TE waves?

Insert the appropriate expressions for r_{12} and r_{23}, and show that the result is the same as $R^E_\perp$.

[B] TM (Transverse magnetic) plane waves are incident on the film with wave vector $\vec{k}_1 = k_{1x}\hat{e}_1 + k_{1z}\hat{e}_3$ as shown in the figure below :



(B1) Write the boundary conditions (4 equations)

(B2) Let $R_{//}^{H} = \dfrac{H_{0R}}{H_{0i}}$ and $T_{//}^{H} = \dfrac{H_{0T}}{H_{0i}}$

i) Eliminate H_{0+} and H_{0-} from the boundary conditions to show that $R_{//}^{H}$ and $T_{//}^{H}$ are related by a transfer matrix $M_{//}^{H}$

$$\begin{bmatrix} 1 \\ \frac{k_{1z}}{n_1 k_1} \end{bmatrix} + \begin{bmatrix} 1 \\ -\frac{k_{1z}}{n_1 k_1} \end{bmatrix} R^H_{//} = M^H_{//} \cdot \begin{bmatrix} 1 \\ \frac{k_{3z}}{n_3 k_3} \end{bmatrix} T^H_{//}\, e^{i k_{3z} L}$$

where the matrix $M^H_{//}$ is given by

$$M^H_{//} = \begin{pmatrix} \cos k_{2z} L & -i \frac{n_2 k_2}{k_{2z}} \sin k_{2z} L \\ -i \frac{k_{2z}}{n_2 k_2} \sin k_{2z} L & \cos k_{2z} L \end{pmatrix}$$

Show that $M^H_{//}$ is unimodular $\det(M^H_{//}) = +1$.

(B3) Solve the boundary condition equations for $R^H_{//}$ to obtain

$$R^H_{//} = - \frac{(n_1^2 k_{3z} - n_3^2 k_{1z}) \cos k_{2z} L + i \left(\frac{k_{3z} k_{1z}}{k_{2z}} n_2^2 - k_{2z} \frac{n_1^2 n_3^2}{n_2^2} \right) \sin k_{2z} L}{(n_1^2 k_{3z} + n_3^2 k_{1z}) \cos k_{2z} L - i \left(\frac{k_{3z} k_{1z}}{k_{2z}} n_2^2 + k_{2z} \frac{n_1^2 n_3^2}{n_2^2} \right) \sin k_{2z} L}$$

what is $R^H_{//}$ for normal incidence $(\vec{k}_1 = k_1 \hat{e}_3)$

$R^H_{//} \rightarrow R_{normal} = ?$ (compare with the result of A.3: $R^E_{\perp} \rightarrow R_{normal}$)).

(B4) Consider $\mathbb{R}$ as given by the multiple reflection calculation (see A.4). What are r_{12} and r_{13} for the oblique incidence TM waves? Insert the appropriate expressions for r_{12} and r_{13} and show that the result is the same as $R^H_{//}$.

[C] An elliptically polarized wave is incident on the film. Incident wave vector is $\vec{k}_1 = a\hat{e}_1 + c\hat{e}_3$,

and the incident electric field is

$$\vec{E}_i = \lambda \left(c\hat{e}_1 + be^{i\phi}\hat{e}_2 - a\hat{e}_3\right) e^{i(\vec{k}_1 \cdot \vec{r} - \omega t)}$$

where a, b, c, ϕ are real constants, λ is a complex constant.

Show that the reflection coefficient is given by:

$$\mathbb{R} = \frac{\left(R_{//} \cdot R_{//}^{*}\right)\left(a^2+c^2\right) + \left(R_{\perp} \cdot R_{\perp}^{*}\right) b^2}{a^2+b^2+c^2}$$

(Note that since $R^H_{//} = R^E_{//}$; we dropped the superscripts from the reflection amplitudes).

Frustrated Total Internal Reflection

[D] Numerical computation

Consider $n_1 = n_3 = 1.52$ (glass)

$n_2 = 1.0$ (vacuum or thin air)

c = velocity of light = 299 792 456 m/sec.

$\lambda_1 = 5.0$ cm (microwave radiation)

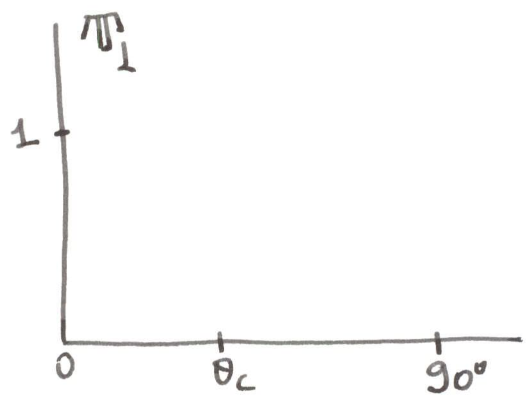
What is the critical angle θ_c for total internal reflection at a glass–air interface?

(D1) Compute transmissivity $\mathbb{T}_\perp$ for TE waves as a function of incident angle θ_i (in degrees) $0° \leq \theta_i \leq 90°$ for the film thicknesses

$$L = \frac{\lambda_2}{10}, \frac{\lambda_2}{4}, \frac{\lambda_2}{2}, \lambda_2$$

$$\mathbb{T}_\perp = 1 - R_\perp \cdot R_\perp^*$$

i.e, plot



(D2) Repeat the same computation for TM waves:

$$\mathbb{T}_{\parallel} = 1 - R_{\parallel} \cdot R_{\parallel}^{*}$$

(D3) Let $\Delta\theta$ be $\Delta\theta = 15°$.

Plot $\mathbb{T}_{\perp}$ and $\mathbb{T}_{\parallel}$ at the incident angles θ_1 and θ_2 as a function of $\frac{L}{\lambda_2}$ in the interval $0 \leq \frac{L}{\lambda_2} \leq \frac{3}{2}$

The incident angles are $\theta_1 = \theta_c - \Delta\theta$, $\theta_2 = \theta_c + \Delta\theta$

(D4) A <u>circularly polarized</u> wave is incident on the air film. Show that the transmissivity is

$$\mathbb{T} = 1 - \frac{1}{2}\left(R_{\perp} \cdot R_{\perp}^{*} + R_{\parallel} \cdot R_{\parallel}^{*}\right)$$

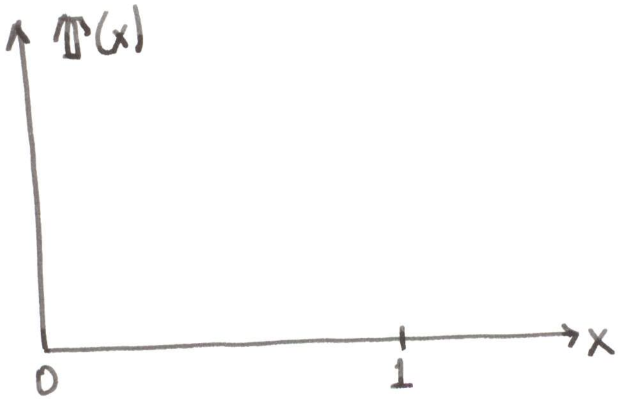
Plot the curves described in (D1) and (D2) for $\mathbb{T}$.

In the general case

$$\mathbb{T}(x) = 1 - \left\{ x\, R_{/\!/} \cdot R_{/\!/}^{*} + (1-x)\, R_{\perp} \cdot R_{\perp}^{*} \right\}$$

What is the meaning of $\mathbb{T}(x)$?

For $\dfrac{L}{\lambda_2} = \dfrac{1}{4}$, plot



for $\theta_1 = \theta_c - \Delta\theta$

$\theta_2 = \theta_c + \Delta\theta$