Problem 1:
Determine if the following series are convergent or not:

a) \[ \sum_{n=2}^{\infty} \frac{(-1)^n}{n-1} \]

b) \[ \sum_{n=1}^{\infty} \frac{n^5}{5^n} \]

c) \[ \sum_{n=0}^{\infty} \frac{n}{(n^2 + 4)^{3/2}} \]

d) \[ \sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!} \]

Problem 2:
Find the interval of convergence of the following series:

a) \[ \sum_{n=0}^{\infty} (-1)^n x^n \]

b) \[ \sum_{n=1}^{\infty} \frac{x^n}{n^2} \]

c) \[ \sum_{n=1}^{\infty} \frac{x^{3n}}{n} \]

d) \[ \sum_{n=1}^{\infty} \frac{(x - 1)^n}{2^n} \]

Problem 3:
Consider a hilly region where the height is defined by

\[ y(x) = 4h \left[ \left( \frac{a}{x} \right)^{12} - \left( \frac{a}{x} \right)^6 \right] \]

as shown in the figure.

When a ball is released at some point very close to the bottom of the valley (meaning \(|x_{\text{initial}} - x_0| \ll a\)). Then, we expect that the ball will move back-and-forth around \(x_0\). Find the frequency of this small oscillation by approximating \(y(x)\) through the Taylor’s expansion around \(x_0\) to 2nd degree, then solving Newton’s equation.

Problem 4:
Prove the following two equations using the infinite series defining sine and cosine functions.

\[ \sin z = \frac{e^{iz} - e^{-iz}}{2i} \]
\[ \cos z = \frac{e^{iz} + e^{-iz}}{2} \]

where \(z = a + iy\) is a complex number.

Problem 5:
Find the absolute value (magnitude) of:

a) \( 2i - 1 \)

b) \( 1 + i \)

c) \( \frac{25}{3 - 4i} \)

Problem 6:
Find the roots of \( f(z) = (z + 1)^4 \).

Problem 7:
Find all values of \( i^{-2i} \).

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Note that potential energy can be defined as \( V(x) = m g y(x) \) which has the form of a potential called as Lennard-Jones potential.