Problem 1: – Problem (1.4.2) of Arfken 4th Ed.
Prove the law of cosines starting from $A^2 = (B - C)^2$

Problem 2: – (1.4.7)
Prove that $(A \times B) \cdot (A \times B) = (AB)^2 - (A \cdot B)^2$.

Problem 3: – (1.5.2)
Verify the expansion of triple vector product
$$A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$$
by direct expansion in cartesian coordinates.

Problem 4: – (1.5.11)
Vector $D$ is a linear combination of three noncoplanar (and nonorthogonal) vectors:
$$D = aA + bB + cC$$
Show that the coefficients are given by a ratio of triple scalar products,
$$a = \frac{D \cdot B \times C}{A \cdot B \times C}, \quad \text{and so on.}$$

Problem 5: – (1.6.5)
Show that $\nabla (uv) = v \nabla u + u \nabla v$, where $u$ and $v$ are differentiable scalar functions of $x, y, \text{and } z$.

Problem 6: – (1.7.5)
Prove $\nabla \cdot (a \times b) = b \cdot \nabla \times a - a \cdot \nabla \times b$
Hint. Treat as a triple scalar product.

Problem 7: – (1.8.17)
The vector potential $A$ of a magnetic dipole, dipole moment $m$, is given by $A(r) = (\mu_0/4\pi)(m \times r/r^3)$.
Show that the magnetic induction $B = \nabla \times A$ is given by
$$B = \frac{\mu_0}{4\pi} \frac{3\hat{r} \cdot \mathbf{m}}{r^3} - \frac{\mathbf{m}}{r^3}$$

Problem 8: – (Restated 1.13.1)
Consider the force $F = r^{2n} \hat{r}$. Find a) $\nabla \cdot F$, b) $\nabla \times F$, c) A scalar potential $\phi(x, y, z)$ so that $F = -\nabla \phi$, d) For what value of the exponent $n$ does the scalar potential diverge at both the origin and infinity?

Problem 9:
Consider the force $F = 2xy \hat{x} + (x^2 + y^2) \hat{y}$. Find the work done by this force while moving from $a = (1, 0, 0)$ to $b = (-1, 0, 0)$ along a semicircular path on the $xy$ plane and passing at $(0, 1, 0)$.

Problem 10:
Suppose $v = (2xy + 3y^2) \hat{y} + (4yz^2) \hat{z}$. Check Stokes’ theorem for the surface shown in the figure.