Problem 1:
Prove the following two equations using the infinite series defining sine and cosine functions.

\[
\sin z = \frac{e^{iz} - e^{-iz}}{2i} \quad \cos z = \frac{e^{iz} + e^{-iz}}{2}
\]

where \(z = x + iy\) is a complex number.

Problem 2:
Find the absolute value (magnitude) of:

a) \(\frac{2i-1}{1-i}\)

b) \(\left(\frac{1+i}{1-i}\right)^5\)

c) \(\frac{25}{3-4i}\)

Problem 3:
Find the roots of \(f(z) = (z + 1)^4\).

Problem 4:
Find all values of \(i^{-2i}\).

Problem 5

a) Find all values of \(\ln z\) where \(z = re^{i\theta}\).

b) Find \(\ln(-x)\) where \(x\) is a positive real number.

Problem 6:
Gamma function is defined as

\[
\Gamma(x) = \int_0^\infty e^{-y}y^{x-1}dy
\]

Evaluate \(\Gamma\left(\frac{1}{2}\right)\). [Hint: Use substitution \(u = \sqrt{y}\).]

Problem 7:
Find the volume of \(n\)-dimensional sphere of radius \(R\), \(V^{(n)}(R) = ?\)

Problem 8:
If Gauss Law is valid in a 4-dimensional space – electric flux through the Gauss surface is equal to \(\rho^{(4)}(\vec{x})/\epsilon_0\), then find the expression for the electric field for a point charge particle with charge \(q\), at a point \(r\) away from the charge.

Problem 9:
Find the following integral with the introduction of \(e^{-\alpha x}\) factor.

\[
I = \int_0^\infty \sin x \frac{1}{x^2 + 1}dx
\]

Problem 10:
Find the following integral.

\[
I = \int_0^\infty \frac{\cos ax}{(1 + x^2)^2}dx
\]