Problem 1:
Consider the following operator:

\[ H = H_0 + H' = \begin{pmatrix} 7 & 24 \\ 24 & -7 \end{pmatrix} + \begin{pmatrix} 0.5 & 1 \\ -2 & 1 \end{pmatrix} \]

Find the 1st and 2nd degree approximations of eigenvalues and 1st degree approximation of eigenvectors using the perturbation theory.

Problem 2:
Consider the following operator

\[ A = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix} \]

a) Find \( e^A \),

b) Find \( \sin(A) \),

c) Find \( \sqrt{A} \), and check if \( \sqrt{A} \cdot \sqrt{A} = A \).

through diagonalization.

Problem 3:
Consider

\[ A = \begin{pmatrix} -4 & 4 \\ -1 & 0 \end{pmatrix} \]

You are to evaluate \( e^A \).

a) Try to evaluate through diagonalization.

b) Set \( A_{22} = \epsilon \) to remove the degeneracy, evaluate \( e^A \) through diagonalization as a function of \( \epsilon \), then find the value at the limit \( \epsilon \to 0 \).

c) Use Cayley-Hamilton theorem to evaluate it directly.