Question 1: (60 pts)

Your friends’ favorite restaurant serves the following meals with the stated calories. \( P(A_i) \) is the probability of being ordered by your friend, which is unknown.

<table>
<thead>
<tr>
<th>Meal</th>
<th>( g(A_i) ) – Energy (kcal)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicken soup</td>
<td>400</td>
<td>( P(A_1) )</td>
</tr>
<tr>
<td>Döner</td>
<td>1700</td>
<td>( P(A_2) )</td>
</tr>
<tr>
<td>Köfte</td>
<td>2000</td>
<td>( P(A_3) )</td>
</tr>
<tr>
<td>Pasta</td>
<td>390</td>
<td>( P(A_4) )</td>
</tr>
<tr>
<td>Pide</td>
<td>1600</td>
<td>( P(A_5) )</td>
</tr>
</tbody>
</table>

You weight your friend after 6 months, and found out that he got 1000 calories per day from this restaurant. You are asked to find the probability distribution of the meals using the principle of maximum entropy.

a) Write down the Lagrangian function \( L \) for this problem (5 pts)

b) Write down the Lagrange multiplier \( \alpha \) in terms of \( \beta \) (5 pts)

c) Write down \( f(b) \) and make a plot of it using Matlab for a reasonable \( b \) range. (20 pts)

d) Find the zero-crossing point by zooming in several times around the region of interest. This is your \( \beta \). Submitting the plot is optional. (Hint: Turn on grids using \texttt{grid} command.) (15 pts)

e) Use \( \beta \) found in previous part, to find \( \alpha \) stated in part (b) (5 pts)

f) Find the probabilities for each meal, \( P(A_i) \), and confirm that: (5 pts)

\[
\sum_i P(A_i) = 1
\]

\[
\sum_i P(A_i)g(A_i) = 1000 \text{ cal}
\]

g) Find entropy for the above probability distribution, and call it \( S_{\text{max}} \) (5 pts)

Note: Do not derive the equations. Use the equations given in class or in the lecture notes.

Question 2: (40 pts)

Consider a mass transportation system with a regular ticket price of 15 TL. They offer \( 2/3 \) discount for the students, and pre-school children (“kids”) can ride for free. The yearly balance sheet shows that the average ticket price per person is 5 TL. If \( P_{\text{regular}} \), \( P_{\text{student}} \), \( P_{\text{kid}} \) represent the probability distribution for these type of the passengers:

a) Write down the constraints for \( P_{\text{regular}} \), \( P_{\text{student}} \), \( P_{\text{kid}} \) as defined above. (8 pts)

b) Find the most likely probability distribution (maximize the entropy.) (8 pts)

c) Find the entropy for the above probability distribution. (8 pts)

d) Now we are told that, they had used a technique to maximize the number of regular passengers. We find out that, according to the constraints given above, \( P_{\text{regular}} = 1/3 \) is the maximum that we can have. (16 pts)

i) Do you expect to have a higher or lower entropy with this additional information?

ii) Find the new probability distribution.

iii) Find the new entropy with the above probability distribution.

e) (Optional) Show that the maximum value of \( P_{\text{regular}} = 1/3 \) using the constraints given above. (8 pts)