Foundation of Newtonian Dynamics: Galilean Relativity

Relativity: Equivalent observers may disagree on details (relative) but they agree on essentials (absolute) and a “dictionary” exists to “translate” the disagreed details. (Empathy)

Galilean Relativity in 1-D: Two equivalent observers O & O’:

\[
\begin{align*}
\{x, t\} & \leftrightarrow \{x’, t’\} ; \quad x \neq x’, \quad t \neq t’ \\
\text{Dictionary:} & \quad x’ = x + U_o t + D_o \\
& \quad t’ = t + T_o
\end{align*}
\]

\[U_o, D_o, T_o : \text{constant!}\]

\[v = \frac{dx}{dt} ; \quad v’ = \frac{dx’}{dt’} = v + U_o \neq v\]
\[ a \equiv \frac{dv}{dt} ; \quad a' \equiv \frac{dv'}{dt'} = a \quad !!! \]

\[ F = m a \quad \text{or} \quad \vec{F} = m \vec{a} \quad \text{(Newton – 2)} \]

\[ \vec{F} = m \vec{a} \quad \text{Not a complete physical law since} \quad \vec{F} \quad \& \quad m \quad \text{are} \]

\[ \text{both new concepts.} \quad \vec{F} \quad \text{should be supplied by other physical laws,} \]

Forces of nature, e.g.: Gravitational, Electromagnetic, Nuclear, ...

Mass \[ [ \text{kg} ] \], Force \[ [ \text{N} = \text{kg m} / \text{s}^2] \]

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FORCES OF FRESHMAN PHYSICS:

1) Gravitational: \[ m g \] (down)

2) External Forces: \[ F \]

3) Tension: \[ T \] (pull only)

4) Spring Forces: \[ -k x \] (k: spring constant)

5) Normal Force: \[ N \] (⊥ to contact surfaces)

6) Friction Force: \[ \vec{f} \] (∥ to contact surfaces)

\[ \vec{f} \uparrow\downarrow \vec{v} \quad \text{in motion,} \quad \vec{f} \uparrow\downarrow \vec{F} \quad \text{when stationary} \]
\( f \) : reactionary force; adjusts itself to cancel all other applied forces when stationary \( (f \leq \mu_s N) \); then somewhat reduced it stays constant \( f = \mu_k N \) after motion starts.

\[ \vec{F} = m \ddot{a} \quad (\text{Newton}) \quad ; \quad \vec{F} + (-m \ddot{a}) = \vec{0} \quad (\text{D’Alambert}) \]

\[ \therefore \text{7) Ghost Force : } (-m \ddot{a}) \text{ reduces all dynamics to statics.} \]

To solve a problem:

1) Draw all real forces: \( m g \), \( F \), \( T \), \( -kx \), \( N \), \( f \)

2) Add the Ghost Force: \(-m \ddot{a}\)

3) Then set \( F_{\text{UP}} = F_{\text{DOWN}} \quad \& \quad F_{\text{RIGHT}} = F_{\text{LEFT}} \)

4) Solve resulting equations.

\[ \vec{F} = m \ddot{a} \]

\[ \vec{F} \rightarrow \vec{0} \quad \Rightarrow \quad \ddot{a} \rightarrow \vec{0} \quad \Rightarrow \quad \vec{v} = \vec{v}_o \quad (\text{Newton - 1}) \]

\( m \): mass at contact point; Let \( m \rightarrow 0 \)

\[ \vec{F}_{\text{on } m} = \vec{0} \rightarrow \vec{F}_{1 \text{ on } m} + \vec{F}_{2 \text{ on } m} = \vec{0} \rightarrow \vec{F}_{12} + \vec{F}_{21} = \vec{0} \]
\[
\begin{align*}
\left| \vec{F}_{12} \right| & = \left| \vec{F}_{21} \right| & \& \hat{F}_{12} & = -\hat{F}_{21} \\
\text{Action} & = \text{Reaction} \quad (\text{Newton} - 3)
\end{align*}
\]

SAMPLE PROBLEMS

1) Free fall
2) Hung mass
3) Conical pendulum
4) Atwood machine → Double Atwood machine
5) Block on table pulled by horizontal force \((\mu = 0)\)
6) Block on table pulled by horizontal force \((\mu = \mu)\)
7) Block on table pulled by hung mass \((\mu = 0)\)
8) Block on table pulled by hung mass \((\mu = \mu)\)
9) Block on inclined plane \((\mu = 0)\)
10) Block on inclined plane \((\mu = \mu)\)
11) Block on inclined plane pulled by hung mass \((\mu = 0)\)
12) Block on inclined plane pulled by hung mass \((\mu = \mu)\)
13) Hung mass in elevator
14) Weight in elevator
15) Atwood machine in elevator
16) Inclined plane in elevator
17) Mass on quarter circle track \((\mu = 0)\)
18) Mass on quarter circle track \((\mu = \mu)\) [Impossible!]

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Dynamics: Given $\vec{F} = \vec{F} (\vec{r}, t)$ find $\vec{r} = \vec{r} (t)$

Too ambitious: Two time integrals !!!

Alternative: Given $\vec{F} = \vec{F} (\vec{r})$ try to find $v = v (\vec{r})$

Simpler: One space integral only.