Physics 201 - Fall 2012 - Electricity and Magnetism

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Textbook: Physics for Scientists and Engineers with Modern Physics

Course webpage: www.phys.boun.edu.tr/~phys201

Lab registration! → www.phys.boun.edu.tr/~physlab

2 Midterms and 1 Final

Phys 201 \(\Rightarrow\) Part I: physics of static charges (electrostatics);
Part II: physics of moving charges.
References
1) Lecture notes by Haluk Bekar
2) "Introduction to Electricity and Magnetism," Liao, Dourmashkin and Belcher.
3) The textbook by Jewett/Serway

Math
1) Vectors
2) Calculus
3) Trigonometry
4) Algebra
Chapter 23: Electric Charges and Fields

**Fundamental Forces:**
1. **Gravitation,** $10^{-34}$, Black holes
2. **Weak,** $10^{-13}$, Neutron decay
3. **Electromagnetism, $10^{-37}$, Atoms, cells, neurons**
4. **Strong, $10^{-39}$, Nuclei**

**Experimental Facts:**
1. Two kinds
2. Local conservation
3. Quantization
4. Inverse square force law
5. Superposition

**Materials:** Insulators, semiconductors, conductors, superconductors

**Coulomb's Law:**
\[
\overrightarrow{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}
\]
- $\overrightarrow{F}_{12}$ is the unit vector that gives the location of $q_1$ relative to $q_2$.
- Force on $q_2$ due to $q_1$.
- Coulomb constant: $k_e = \frac{1}{4\pi \varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$

\[
|\overrightarrow{F}_{12}| = k_e \frac{|q_1||q_2|}{r^2} : \text{magnitude of the electrostatic force.}
\]

**Hydrogen Atom:**
\[
F_e = k_e \frac{|e||e|}{r^2} = 8 \times 10^{-8} \text{ N}
\]
\[
F_g = G \frac{m_e m_e}{r^2} = 3.6 \times 10^{-47} \text{ N}
\]
\[
\Rightarrow \frac{F_e}{F_g} \approx 2.4 \times 10^{39}
\]

**Charges are not moving!!**

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Scalar fields, vector fields.

**The Electric Field:**
- gravitational force
- electric force

gravitational force is always attractive; \( G = 6.67 \times 10^{-11} \text{ N.m}^2/\text{kg}^2 \)

gravitational constant kg

non-contact forces!

“action at a distance” type of force

such forces are carried by fields. \( \Rightarrow \) “action by continuous contact”.

\[
\vec{F}_g = -G \frac{m_1 m_2}{r^2} \hat{r}
\]

\( \hat{r} \) unit vector

\( r \) is a contact force

\[
\vec{F}_g = -G \frac{m_1 m_2 \hat{r}}{r^2}
\]

Consider a small mass very near the surface of Earth.

\[
|\vec{F}_g| = G \frac{m \text{earth} m}{r_{\text{earth}}^2} = m \frac{G \text{earth}}{r_{\text{earth}}^2} \Rightarrow |\vec{F}_g| = \frac{G \text{earth}}{r_{\text{earth}}^2}
\]

so \( m \) is called “test mass.”

Therefore, \( \vec{g} = \frac{G \text{earth}}{r_{\text{earth}}^2} \) and \( |\vec{g}| = 9.8 \text{ N/kg} \) near the surface of Earth.

gravitational field \( \Rightarrow \) test mass (very small mass compared with the mass of Earth).

**The Electric Field:**

\[
|\vec{F}_e| = k \frac{q_0 \vec{q}}{r^2} = k \frac{q_0}{r^2} \vec{E} = q_0 |\vec{E}|
\]

\( k = \frac{1}{4\pi \epsilon_0} \)

\( \vec{E} \): electric field; unit of \( \vec{E} \) is \( \frac{\text{N}}{\text{C}} \)

Two charges are needed to quantify the electric force.

Electric field exists in the region of space around a charge or a charged object. \( \vec{E} \) and \( \vec{g} \) are vector fields!!!
Test charge \( q_0 \) is always (+).

If the source charge \( q \) is (+) then \( \vec{E} \) is directed away from \( q \).

If the source charge \( q \) is (-) then \( \vec{E} \) is directed toward \( q \).

Electric field due to a finite number of point charges => principle of superposition

\[ \vec{E} = k_e \sum \frac{q_i \hat{r}_i}{r_i^2} \]

Electric field of a continuous charge distribution:

\[ \Delta \vec{E} = k_e \frac{\Delta q}{r^2} \hat{r} \Rightarrow \vec{E}_{\text{Total}} \approx k_e \sum \frac{\Delta q_i \hat{r}_i}{r_i^2} \]

\[ \vec{E} = k_e \lim_{\Delta q \to 0} \sum i \frac{\Delta q_i \hat{r}_i}{r_i^2} = k_e \int \frac{dq \hat{r}}{r^2} \]

Integration is over entire charge distribution.

\[ \vec{E} = k_e \int \frac{dq}{r^2} \hat{r} \] and note: \( \hat{r} = \frac{\hat{r}}{r} \)

**Linear charge density:** \( \lambda = \frac{q}{L} \)

**Surface charge density:** \( \sigma = \frac{Q}{A} \)

**Volume charge density:** \( \rho = \frac{Q}{V} \)

Uniform charge distribution \( \Rightarrow \lambda = \frac{Q}{L} \); \( \sigma = \frac{Q}{A} \); \( \rho = \frac{Q}{V} \)

Nonlinear charge distribution \( \Rightarrow \lambda = \frac{dq}{dt} \); \( \sigma = \frac{dq}{dA} \); \( \rho = \frac{dq}{dV} \)
How do we represent an electric field?

1. vector field representation; 2. field lines.

Electric field lines is a way to represent the collection of vectors that constitute the field.

There are two rules to draw E-field lines:

1) The direction of the field line at any point in space is tangent to the field at that point.

2) The field lines never cross each other, otherwise there would be two different field direction at the point of intersection.

Electric field lines always start from a positive charge and end on a negative charge.
Uniform E  
(field lines are parallel)

Non-uniform E  
(field lines are not parallel)

Weak E  
(field lines are less dense)

Strong E  
(field lines are dense)

density of the field lines is proportional to the strength of E.

Not possible! Field lines do not cross each other!

\[ \vec{E} \]  
field line

\[ \vec{E} \] is tangent to the field line.
Electric Field by a Line of Charge

Step 1:
- Circular: \( dq = \lambda ds \)
- Straight: \( dq = \lambda dx \)

Step 2:
Write \( d\vec{E} \) using \( \lambda dx \) or \( \lambda ds \)
If the charge is (-), \( d\vec{E} \) points toward \( dq \).
If the charge is (+), \( d\vec{E} \) points away from \( dq \).

Step 3:
Look for symmetry.
1. Straight line, 2. Ring, 3. Circular arc

Step 4:
Do the integration. If you change the integration limits, the sign would change. Discard the minus sign. Replace \( \lambda \) with \( \frac{Q}{L} \).
1) \( \vec{E} \) of a uniformly charged rod along axis:

\[
\vec{E} = \int d\vec{E} \quad (-\hat{i})
\]

and \( d\vec{E} = k \frac{dq}{x^2} \Rightarrow E = k \int \frac{dq}{x^2} = k \int_a^{l+a} \frac{\lambda dx}{x^2} = \frac{k \lambda l}{a(l+a)} = \frac{kQ}{a(l+a)} \hat{i}
\]

\[\text{[Electric field is a vector]} \Rightarrow \vec{E} = k \frac{Q}{a(l+a)} \quad (-\hat{i})\]

what if \( \lambda \neq \text{constant} \)?

For away from the rod \( a \gg l \Rightarrow \vec{E} = -k \frac{Q}{a^2} \hat{i} \): looks like a point charge.

\[\text{\( \Rightarrow a \gg l \Rightarrow a, l+a < a \)}\]
2) E of a uniformly charged rod along perpendicular bisector:

\[ d \vec{E} = d E_x \hat{i} + d E_y \hat{j} \]
\[ = -d E \sin \theta \hat{i} + d E \cos \theta \hat{j} \]

due to symmetry \( \int d E_x = 0 \)

\[ dq = \lambda \, dx \quad r = \sqrt{y^2 + x^2} \quad \cos \theta = \frac{y}{r} = \frac{y}{\sqrt{y^2 + x^2}} \]

\[ E_y = \int d E_y = \int d E \cos \theta \]

\[ E_y = k \lambda \int_{-l/2}^{l/2} \frac{dx}{r^2} \frac{y}{r} = k \lambda y \int_{-l/2}^{l/2} \frac{dx}{(y^2 + x^2)^{3/2}} \]

\[ x = y \tan \theta \Rightarrow dx = y \sec^2 \theta \, d\theta \quad ; \quad x = -l/2 \Rightarrow \theta = -\theta_0 \]
\[ x = l/2 \Rightarrow \theta = \theta_0 \]

\[ \Rightarrow E_y = k \lambda y \int_{-\theta_0}^{\theta_0} \frac{y \sec^2 \theta \, d\theta}{y^3 (\tan^2 \theta + 1)^{3/2}} = \frac{k \lambda}{y} \int_{-\theta_0}^{\theta_0} \frac{\sec^2 \theta \, d\theta}{\sec^3 \theta} \]

\[ \Rightarrow E_y = \frac{k \lambda}{y} \int_{-\theta_0}^{\theta_0} \cos \theta \, d\theta \Rightarrow E_y = \frac{2k \lambda \sin \theta_0}{y} \]

and note that \( \sin \theta_0 = \frac{l/2}{y^2 + (l/2)^2} \)
\[
E_y = \frac{2 k \lambda \sin \theta_0}{y} \quad \text{limiting case: } \theta_0 \to 90^\circ \implies E_y = \frac{2 k \lambda}{y}, \text{ infinite rad}
\]

or equivalently write \(\sin \theta_0 = \frac{1/2}{\sqrt{y^2 + (1/2)^2}}\)

\[
\implies E_y = \frac{2 k \lambda}{y} \cdot \frac{1/2}{\sqrt{y^2 + (1/2)^2}} \cdot \sqrt{y^2 + (1/2)^2} = \frac{q}{y}
\]

then the limiting cases:
1) \(y \gg 1 \implies \text{point charge limit } E_y = \frac{k q}{y^2}\)
2) \(l \gg y \implies E_y \approx \frac{2 k \lambda}{y}, \text{ infinite rad}\)
3) E of a ring along axis

Two components: $dE \cos \theta$, $dE \sin \theta$

Due to symmetry $\int dE \sin \theta = 0$.

$dE_2 = dE \cos \theta = k \frac{dq}{r^2} \cos \theta$

From geometry $\cos \theta = \frac{r}{r} \Rightarrow dE_2 = k \frac{dq}{r^2} \frac{2\pi r}{2\pi}$

$\Rightarrow d\theta = \lambda ds \Rightarrow E_2 = k \lambda \int_0^{2\pi} \frac{2\pi r}{r^2} ds = k \frac{2\pi \lambda}{r^2} \int_0^{2\pi} ds$

$\Rightarrow E_2 = k \frac{2\pi \lambda}{r^2} \frac{2\pi R}{(2^2 + R^2)^{3/2}}$

and note that $\lambda 2\pi R = q$

$\Rightarrow E_2 = \frac{9}{4\pi \varepsilon_0} \frac{2}{(2^2 + R^2)^{3/2}}$

Limiting cases: 1) $\gg \Rightarrow E_2 = \frac{1}{4\pi \varepsilon_0} \frac{q}{2^2}$: ring looks like a point charge far away from the ring!
4) E of a disk along axis

\[ dq = \delta dA = \delta \left( 2\pi rd\!dr \right) \]

\[ \delta E_2 = dq \cdot \frac{\varepsilon}{4\pi \varepsilon_0 (z^2+r^2)^{3/2}} = \frac{\varepsilon}{4\pi \varepsilon_0} \frac{2\pi r dr}{(z^2+r^2)^{3/2}} \]

\[ \Rightarrow E_2 = \frac{\delta z}{4\varepsilon_0} \int_0^R \frac{2r dr}{(z^2+r^2)^{3/2}} \]

\[ u = z^2 + r^2 \Rightarrow du = 2r dr \quad ; \quad r=0 \Rightarrow u=z^2 \quad ; \quad r=R \Rightarrow u=z^2+R^2 \]

\[ \Rightarrow E_2 = \frac{\delta z}{4\varepsilon_0} \int_0^R \frac{z^2+r^2}{u^{3/2}} du \]

Note: \[ \int u^{-3/2} du = -2u^{-1/2} \]

\[ \Rightarrow E_2 = \frac{\delta z}{4\varepsilon_0} \left( -2u^{-1/2} \right) \frac{z^2+r^2}{z^2} \]

\[ = \frac{\delta z}{2\varepsilon_0} \left( \frac{1}{2} - \frac{1}{\sqrt{z^2+r^2}} \right) \]

\[ E_2 = \frac{\delta}{2\varepsilon_0} \left( 1 - \frac{z}{\sqrt{z^2+r^2}} \right) \quad z > 0 \]

Limiting cases: 1) \( R \to \infty \Rightarrow \) \( E = \frac{\delta}{2\varepsilon_0} \) : infinite sheet of charge

\[ \uparrow \text{ very important} \]

5) E of a punctured disk (annulus) along axis: Homework!

Hint: Integration is from \( R_1 \) (inner radius) to \( R_2 \) (outer radius)
6) $E$ of a square along perpendicular axis: **Homework!**

7) $E$ of a "dipole" rod along perpendicular axis: **Homework!**

8) $E$ of a compound rod along perpendicular bisector by superposition: **HW**
A line of charge is bent into a semicircle. Charge distribution is NOT UNIFORM.

A line of charge is formed into a semicircle of radius $R$ as shown in the figure. The charge per unit length along the semicircle is described by the expression $\lambda = \lambda_0 \cos \theta$. Calculate the total force vector on a point charge $q$ placed at the center of curvature $P$.

\[
\int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} \quad \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} \quad \int \sin x \cos x \, dx = \frac{\sin^2 x}{2}
\]

$\vec{E}$ is in the $-x$ direction alone due to symmetry.

\[
d\vec{E} = k \frac{dq}{R^2}
\]

\[
dE_x = -k \frac{dq}{R^2} \cdot \cos \theta = -k \lambda_0 R \cos \theta \, d\theta = - \frac{k \lambda_0}{R} \cos \theta
\]

\[
E_x = \int_{-\pi/2}^{\pi/2} dE_x = -k \frac{\lambda_0}{R} \left[ -\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_{-\pi/2}^{\pi/2} = - \frac{k \lambda_0 \pi}{2R}
\]

\[\Rightarrow \vec{E} = -\frac{k \lambda_0 \pi}{2R} \uparrow \text{ thus } \vec{F} = q \vec{E} = -\frac{k \lambda_0 \pi q}{2R} \uparrow\]
Electric Dipoles: Neutral structure composed of two nearby, equal but opposite charges.

\[ \vec{E} = \vec{E}_1 + \vec{E}_2 \quad ; \quad E_x = E_{1x} + E_{2x} \quad ; \quad E_y = E_{1y} + E_{2y} \quad ; \quad E_y = 0 \, . \]

\[ E_x = \frac{2kq}{a^2 + y^2} \, \cos \theta \]

From geometry \( \cos \theta = \frac{a}{r} = \frac{a}{\sqrt{a^2 + y^2}} \),

\[ \Rightarrow E_x = \frac{2kqa}{(a^2 + y^2)^{3/2}} \]

**Important limiting cases:** \( y \gg a \Rightarrow E \propto \frac{k2aq}{y^3} \) at large distances.

- Point charge \( E \propto \frac{1}{r^2} \)
- Electric dipole \( E \propto \frac{1}{r^3} \)
- Line of charge \( E \propto \frac{1}{r} \)
- Plane of charge \( E \) is constant.

Water molecules are dipoles!