Chapter 20: Sources of the magnetic field

A moving charge ① experiences a force in a magnetic field.
② creates a magnetic field.

\[ \mathbf{B} = \frac{\mu_0}{4\pi} \frac{q \mathbf{v} \times \hat{r}}{r^2} \]

**magnetic field of a point charge with constant velocity.**

\( \mathbf{B} \) at \( P_2 \) = 0. \( \mathbf{B} \) at \( P_1 \) is into the page.

**Right Hand Rule**

(a) Magnetic-field vectors due to a moving positive point charge \( q \). At each point, \( \mathbf{B} \) is perpendicular to the plane of \( \mathbf{r} \) and \( \mathbf{v} \), and its magnitude is proportional to the sine of the angle between them. (b) Magnetic field lines in a plane containing a moving positive charge.

The magnetic field of (a) a moving positive charge, and (b) a moving negative charge, when the speed of the charge is small compared to the speed of light.

A moving positive charge also produces an electric field. Electric field lines radiate outward from the positive charge.

The magnetic field lines are circles centered on the line of \( \mathbf{v} \) and lying in planes perpendicular to this line. Field line direction is found by the right hand rule.

\( \mu_0 \): permeability of free space

\[ \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \]
Magnetic field of a current element

A single moving charge will not generate a steady magnetic field. Stationary charges generate steady electric fields. Steady currents generate steady magnetic fields. The total magnetic field caused by several moving charges is the vector sum of the fields caused by the individual charges. Principle of superposition.

\[ \hat{d}\mathbf{B}_{\text{out}} \]
\[ \hat{d}\mathbf{B}_{\text{in}} \]

\[ \hat{r} \]
\[ \hat{r} \]
\[ \theta \]
\[ d\mathbf{r} \]
\[ P \]
\[ P' \]

\[ d\mathbf{B} = \frac{\mu_0 dq \hat{v} \times \hat{r}}{4\pi r^2} \]

Note: \( dq \hat{v} = dq \frac{d\mathbf{s}}{dt} = \frac{dq}{dt} d\mathbf{s} \)

\[ \Rightarrow d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{s} \times \hat{r}}{r^2} \]

Biot-Savart Law

Total magnetic field of the current wire:

\[ \mathbf{B} = \int_{\text{wire}} d\mathbf{B} = \int_{\text{entire wire}} \frac{\mu_0}{4\pi} \frac{I d\mathbf{s} \times \hat{r}}{r^2} \]

Electric field \( \leftrightarrow \) Coulomb's law. \( \leftrightarrow \) \( dq \)
Magnetic field \( \leftrightarrow \) Biot-Savart law. \( \leftrightarrow \) \( I d\mathbf{s} \)
Example: \( \mathbf{B} \) of a very long wire \((L \to \infty)\)

\[
\mathbf{B}(p) = \mathbf{?} \quad \text{note: } |\mathbf{r}| = r \quad \text{and} \quad \hat{r} = \frac{\mathbf{r}}{r}
\]

\[
d\mathbf{B} = \frac{\mu_0 I}{4\pi} \frac{ds \times \hat{r}}{r^2}, \quad ds = I dy \hat{r}, \quad \hat{r} = \sin\theta \hat{i} + \cos\theta \hat{j}
\]

\[
\Rightarrow d\mathbf{B} = \frac{\mu_0 I}{4\pi} \frac{dy \sin\theta (-\hat{k})}{r^2} \quad \text{since } I dy \hat{r} \times (\sin\theta \hat{i} + \cos\theta \hat{j}) = I dy \sin\theta (-\hat{k})
\]

\[
\therefore \mathbf{B} = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{dy \sin\theta (-\hat{k})}{r^2} \quad \text{where } r^2 = R^2 + y^2
\]

\[
(-) \quad \text{since } y < 0
\]

\(y\) and \(\theta\) are not independent: from the geometry \(y = -\frac{R}{\tan\theta}\)

\[
\Rightarrow y = -R \cot\theta \quad \Rightarrow dy = R \csc^2 \theta \, d\theta = \frac{R \, d\theta}{\sin^2 \theta} = \frac{R \, d\theta}{(R/r)^2} = \frac{R}{R}
\]

\(\text{note that } y = -\infty \text{ corresponds to } \theta = 0\).

\(y = \infty \text{ corresponds to } \theta = \pi \text{ radians}. \quad (\text{since } y = -R \cot\theta).\)

\[
\Rightarrow \mathbf{B} = -\frac{\mu_0 I}{4\pi R} \int_0^{\pi} \sin\theta \, d\theta \hat{k} = \frac{\mu_0 I}{4\pi R} \left[ \hat{k} \right]_0^{\pi} = -\frac{\mu_0 I \hat{k}}{2\pi R}
\]

\[
\mathbf{B} = -\frac{\mu_0 I}{2\pi R} \hat{k} , \quad \mathbf{B} = \frac{\mu_0 I}{2\pi R} \text{ magnetic field of an infinite current wire}
\]