Chapter 2

Limits and Continuity

The average rate change of \( y(x) = f(x) \) over interval \([x_1, x_2]\) is

\[
\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}, \quad h = x_2 - x_1
\]

slope = \( m \)
\[
\frac{\Delta y}{\Delta x}
\]

Slope of the line (secant) thru P and Q

Slope of a line

\[
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[y = mx + b\]
\[ y(2+h) = f(2+h) = (2+h)^2 \]

Slope of the secant = \( \frac{\Delta y}{\Delta x} \)

\[ \frac{\Delta y}{\Delta x} = \frac{f(2+h) - f(2)}{2+h - 2} \]

\[ f(2) = 2^2 = 4 \]

\[ f(2+h) = (2+h)^2 = 4 + 2h + h^2 \]
\[
\frac{\Delta y}{\Delta x} = \frac{f(2+h) - f(2)}{2+h-2} = \frac{4+4h+h^2 - 4}{h(2+h)} = \frac{h}{h(2+h)} = 4+h
\]

When the secant line touches the curve at a single point, it becomes tangent.
The slope of the tangent line at \( x = 2 \), that is \( h \to 0 \), is

\[
\frac{\Delta y}{\Delta x} = 4 + h = 4
\]

Tangent means the slope of a single point on the graph (wave).
The slope of the tangent line to \( y(x) = f(x) \)

corresponding point \( P(x_0, y_0) \) is

\[
\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(x_0) = \left( \frac{dy}{dx} \right)_{x = x_0}
\]

derivative of \( f(x) \) at \( x_0 \).
\[ f'(x_0) \text{ means the instantaneous rate of change of } f(x) \text{ at } x_0. \]

\[ \frac{df}{dx} \bigg|_{x_0} = f'(x_0) \]

\[ \frac{\Delta y}{\Delta x} \text{ is the average rate of change of } f(x) \]

**Ex**

\[ y = mx + b \rightarrow y' = m \]

\[ y = c \rightarrow y' = 0 \]