Istanbul Şehir University
Math 104

Date: 5 April 2014
Time: 10:00-11:30
Full Name:
Student ID:
Spring 2014 Second Exam

IMPORTANT
1. Write down your name and surname on top of each page. 2. The exam consists of 4 questions, some of which have multiple parts. 3. Read each question carefully and put your answers neatly on the answer sheets. Simplify your answers. 4. Show all your work. Correct answers without justification will not get credit. 5. Unless otherwise specified, you may use any method from classwork to solve the problems. 6. Calculators are not allowed. 7. All cellphones and electronic devices are to be kept shut and out of sight.

<table>
<thead>
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<th>Q1</th>
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<tr>
<td>20 pts</td>
<td>30 pts</td>
<td>30 pts</td>
<td>20 pts</td>
<td>100 pts</td>
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Q1. Evaluate the improper integral

\[
\int_{1}^{\infty} \frac{dx}{\sqrt{x(x+1)}}
\]

Let \( u = \sqrt{x} \Rightarrow u^2 = x \)

\[
2u\ du = dx
\]

\( x = 1 \Rightarrow u = 1 \), \( x = \infty \Rightarrow u = \infty \)

\[
\left[ \int_{1}^{\infty} \frac{2u\ du}{u(u^2 + 1)} \right]_{1}^{\infty}
\]

\[
= 2 \arctan u \bigg|_{1}^{\infty}
\]

\[
= 2 \lim_{b \to \infty} \arctan b - 2 \arctan 1
\]

\[
= 2 \cdot \frac{\pi}{2} - 2 \cdot \frac{\pi}{4}
\]

\[
= 2 \cdot \frac{\pi}{4} = \frac{\pi}{2}
\]
Q2. Evaluate the following integrals:

(a) \( \int \frac{dx}{x^2+4x} \) 

Partial fractions:
\[
\frac{1}{x^2+4x} = \frac{1}{x(x+4)} = \frac{A}{x} + \frac{B}{x+4}
\]

\[
A(x+4) + Bx = 1
\]

At \( x = 0 \):
\[
1 = 4A \quad A = \frac{1}{4} = -B
\]

At \( x = -4 \):
\[
1 = -4B
\]

\[
\int \frac{dx}{x^2+4x} = \frac{1}{4} \left( \int \frac{dx}{x} - \frac{1}{4} \int \frac{dx}{x+4} \right)
\]

\[
= \frac{1}{4} \ln |x| - \frac{1}{4} \ln |x+4| + C
\]

(b) \( \int \tan^{-3/4} \theta \sec^3 \theta d\theta \)

\[
\sec^3 \theta = \frac{\sec^2 \theta}{1 + \tan^2 \theta}
\]

\[
= \int \tan^{-3/4} \theta \sec^2 \theta d\theta + \int \tan^{5/4} \theta \sec^2 \theta d\theta
\]

Let \( u = \tan \theta \Rightarrow du = \sec^2 \theta d\theta \)

\[
= \int u^{-3/4} du + \int u^{5/4} du
\]

\[
= \frac{1}{\text{11/4}} u^{11/4} + \frac{u^{9/4}}{9/4} + C
\]

\[
= \frac{11}{11/4} u^{11/4} + \frac{9}{9/4} u^{9/4} + C
\]

\[
= 4 \tan \theta + \frac{9}{9} \tan \theta + C
\]
Q3. Evaluate the following integrals:

(a) \[ \int x \sin x \, dx \]
    \[
    u = x \quad dv = \sin x \, dx \\
    du = dx \quad v = -\cos x \\
    \]
    \[
    \therefore \int x \sin x \, dx = -x \cos x + \int \cos x \, dx \\
    = -x \cos x + \sin x + C
    \]

(b) \[ \int x^{-2/3} \ln x \, dx \]
    \[
    u = \ln x \quad dv = x^{-2/3} \, dx \\
    du = \frac{dx}{x} \quad v = x^{1/3} = 3x^{1/3} \\
    \]
    \[
    \therefore \int x^{-2/3} \ln x \, dx = 3x^{1/3} \ln x - 3 \int \frac{x^{1/3}}{x} \, dx \\
    = 3x^{1/3} \ln x - 3 \int x^{-2/3} \, dx \\
    = 3x^{1/3} \ln x - 3 \cdot \frac{x^{1/3}}{1/3} + C \\
    = 3x^{1/3} \ln x - 9x^{1/3} + C
    \]
Q4. Evaluate the following integral:

\[ \int \frac{dx}{(x^2 + 1)^{3/2}} \]

**Trig Substitution:**

\[ x = \tan \theta \]
\[ \sec^2 \theta \, d\theta = \, dx \]
\[ \sec \theta = \sqrt{x^2 + 1} \]

\[ L^2 = \int \frac{\sec^2 \theta \, d\theta}{\sec^3 \theta} \]

\[ = \int \frac{d\theta}{\sec \theta} \]

\[ = \int \cos \theta \, d\theta = \sin \theta + C \]

\[ = \frac{x}{\sqrt{x^2 + 1}} + C \]