Center of gravity

A point in a body around which the net torque due to gravity is zero.

If you can support an object at its c.o.g., it won’t topple.

\[ \sum F_{g} \times d \] torques due to gravity

Total force due to gravity & gravitational field.

\[ \sum m_{i} g \times x_{i} = \sum m_{i} \times x_{i} \]

Remember: Center of mass:

\[ \mathbf{r}_{cm} = \frac{\sum m_{i} \mathbf{r}_{i}}{\sum m_{i}} = \frac{\mathbf{m}_{rot} \times \mathbf{a}_{cm}}{\mathbf{r}_{cm} \cdot \mathbf{a}_{cm}} \]

Continuous:

\[ \mathbf{r}_{cm} = \frac{\int \mathbf{r} \, dm}{\int dm} \] rigid objects

Rigid body:: Collection of particles at fixed position with respect to each other.

This is an idealization, special relativity does not allow idealized rigid bodies.

Rigid bodies can translate and/or rotate.

Equilibrium:

\[ \sum F_{ext,i} = 0 \]

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Sum of all forces with respect to some point P.

Static equilibrium: In the frame that we analyze the system, it has \( \mathbf{v} = 0 \) & \( \mathbf{a} = 0 \).

Torque:

\[ \mathbf{\Sigma F} \times \mathbf{r} \]

Lagrange from \( \mathbf{F} + \mathbf{F} \).

Note that the torque is dependent on where we choose as the pivot.

\[ \mathbf{\Sigma F}_{r} = \sum \mathbf{F}_{i} \times \mathbf{r}_{i} \]

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So, if \( \mathbf{\Sigma F} = 0 \) then \( \mathbf{\Sigma F}_{r} = \mathbf{\Sigma F}_{r} \)

If we have \( \mathbf{\Sigma F} = 0 \) for a set of forces, then their torque with respect any point is the same. If \( \mathbf{\Sigma F}_{r} = 0 \), static equilibrium.

\[ \text{Example 12.5 in Zemansky 8th edition.} \]

When there is no person on the ladder, what is the minimum angle \( \theta \) such that the ladder does not slide.

(Assume the friction between the ladder & the wall is negligible.)

\[ N_{w} = F_{g} = \frac{N_{l}}{2m} \]

\[ \theta = \tan^{-1} \frac{1}{2m} \]

\[ \text{When} \theta = 1, \text{what happens?} \]

\[ \text{When} \theta = 2, \text{what happens?} \]