Force: Any interaction that tends to change the motion of an object. All of the forces are based on four fundamental interactions: electromagnetism, weak & strong nuclear interactions & gravity.

How to measure forces in the lab?
La simplest method is to use a spring scale. Force is directly proportional to the extension of the spring. 1675-Hooke's Law:
\[ F = -kx, \text{ "ut teno, sic vis!"} \]

Hooke's contemporary Isaac Newton published Philosophiae Naturalis Principia Mathematica in 1687. Three laws of motion:
1. Every object remains in a state of constant velocity if there are no forces acting on it.
   - Observing in an inertial frame.
2. If the vector sum of all the forces on a body is \( \vec{F} \), then \( \vec{F} = m \vec{a} \). An inertial mass is simply defined as the proportionality constant between force & acceleration.
3. When an object exerts a force \( \vec{F} \) on a second body, the second body exerts a force \( -\vec{F} \) on the first body. (Action-reaction)

1 unit of force is Newton = kg m/s².

Weight of an object is defined as the force it experiences due to gravity. So the weight of the object is dependent on whether one is on the surface of Earth, or the Moon, etc. spherical.

The gravitational force between any two bodies is given by \( F_g = \frac{G M_1 m_2}{R^2} \) where \( M_1 \) and \( m_2 \) are the gravitational masses of the objects, \( d \) the distance between their centers and \( G \) is the gravitational constant.

For a little ball of gravitational mass \( m \), the force then becomes:
\[ F_g = \frac{G m M_e}{R_e^2} \]

where \( R_e \) is the radius of Earth & \( M_e \) is the mass of Earth.

As \( h \ll R_e \), we can write this:
\[ F_g \approx \frac{G M_e}{R_e^2} m = 6.67 \times 10^{-11} \text{ N}(m/kg)^2 \]

\[ \frac{G M_e}{R_e^2} = 9.82 \text{ m/s}^2 \]

\[ M_e = 5.97 \times 10^{24} \text{ kg} \]

\[ g = \text{gravitational acceleration} \]

So on the surface of the Earth we can approximately say that \( F_g = m g \) where \( m \) is the gravitational mass of the object.

But why is the gravitational mass the same as the inertial mass (up to a scaling factor)? We know the hammer & feather experiment, that should mean that: \( pm = pma \Rightarrow \text{acceleration is independent of the object.} \)

Why do the two m's cancel? Why is it that the quantity/property of an object that determines how much it is attracted to other objects turns out to be the same as the proportionality constant that determines how much it can be accelerated?

In 1889, Loránd Eötvös measured the two masses to be within \( \frac{1}{2000000} \) of each other. Further experiments steadily improved this precision.

Explanation by A.Einstein in General Relativity.

Side question: How do objects like Earth and the Sun that are hundreds of kilometers apart pull each other? Is the force instantaneous?

To be discussed later.

Now let us look at some applications of Newton's laws of motion. For this, we will start by defining a few words:

- **Tension**: Pulling force exerted by each end of a string, chain, etc. Tension is not a force, but has the same units.

Defining tension makes sense in the light of Newton's third law in particular.

Each segment/strand of the chain is subject to Newton's laws.

- \( F_{1i} \) : force of the 2nd segment on the 1st segment.
- \( F_{2i} \) : force of the 3rd segment on the 2nd segment.
- \( \vec{F}_1 \) : force on the 1st segment.
- \( \vec{F}_2 \) : force on the 2nd segment.
- \( \vec{F}_3 \) : force on the 3rd segment.

\[ \text{Net Force} = \vec{F}_x \]

\[ \text{The spot of tang-\_ wander:} \]

\[ \boxed{\text{Ex}} \]

\[ \boxed{\text{F}_{12} \text{force of the 2nd segment on the 1st segment}} \]

\[ \boxed{\text{F}_{23} \text{force of the 3rd segment on the 2nd segment}} \]

\[ \boxed{\text{F}_{13} \text{force of the 1st segment on the 2nd segment}} \]
Normal force: When objects make contact, there exist forces on each other. The component of this contact force that is perpendicular to the contact surface is called the normal force. The horizontal component is "friction" as surfaces will try to keep contact and any sliding, i.e., the horizontal motion is objected.

Consider we have two objects on a frictionless surface connected to each other with a weightless rod. The maximum tension that the rod can hold is $T_{max}$.

The second object, of mass $m_2$, is being pulled horizontally by some external force $F_{ext}$.

What is the maximum possible acceleration of the system if we don’t want the rod to yield? What is the maximum possible $F_{ext}$?

While these problems look “crafted,” they can easily pertain to real life. Consider two train cars connected. The friction will be small (thanks to the wheels) and the rod connecting the many-tonne cars can indeed be modelled as weightless compared to the cars. $F_{ext}$ could be the force from the locomotive.

Free-body diagram for $m_2$:

\[ F_{max} = (m_1 + m_2) \cdot a_{max} = \frac{T_{max}}{m_1} \]

The two cars should have the same speed at all times to make sure that the length of the rod is not change, so they should have the same acceleration. Hence: $F_{ext} = (m_1 + m_2) \cdot \frac{T_{max}}{m_1}$

Ex 2/ What would happen if the train was going up a ramp?