Case 1: The car is initially stationary. In this case, the net force is $F = \tau / r$.

What happens when the car is in motion? The car will experience a centripetal force, $F = m \frac{v^2}{r}$.

Case 2: The car is moving on a circular track. The centripetal force is $F = m \frac{v^2}{r}$.

What happens if the car is accelerating? The net force is $F = m \frac{v^2}{r} + F_{net}$.

Note that if the car is accelerating, $v^2 = r(a + N)$.

Case 3: The car is performing a smooth, circular motion. The net force is $F = m \frac{v^2}{r} + F_{net}$.

Acceleration is $a = \frac{V^2}{L}$.
Banked Road—Continued:

Case 2—With friction, U.C.M.

Subcase 2: \( v > \sqrt{gR \tan \theta} \)

In this subcase, the car would be slipping "upward" if there were no friction. So \( F_p \) should point along the -x direction. The car is in U.C.M. so the net acceleration should be: \(-v^2 \frac{R}{L}\).

\[ a_y = 0 \Rightarrow N \cos \theta - F_p \sin \theta - mg = 0 \]
\[ a_x = \frac{v^2}{R} \Rightarrow -N \sin \theta - F_p \cos \theta = -m \frac{v^2}{R} \]

We know \( F_p \leq \mu_s N \), so we can ask what the maximum safe speed is.

\[ F_p \leq \mu_s N \]
\[ N \sin \theta + F_p \cos \theta \leq N \sin \theta + \mu_s N \cos \theta \]
\[ \frac{v^2}{R} \leq N (\sin \theta + \mu_s \cos \theta) \]

Hence we find that the max. allowed speed is:

\[ v \leq \sqrt{gR \frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta}} \]

- If we are on a flat road (no banking), \( \theta = 0 \):

\[ v \leq \sqrt{gR \frac{0 + \mu_s}{1 - \mu_s}} \]

This is exactly the same answer we got for the flat road.

- We also note that in the absence of friction, i.e., when \( \mu_s = 0 \), \( v \leq \sqrt{gR \tan \theta} \), so there is only one allowed speed for that scenario.

Wall of Death: When \( \theta = \frac{\pi}{2} \), the subcase 1 of case 2 becomes highly interesting. We essentially have a car in the vertical direction! Using the formula we had derived earlier:

\[ v \geq \sqrt{gR \frac{1 - \mu_s \cos \theta}{\cos \theta + \mu_s \sin \theta}} \]

We find that if we are fast enough we can successfully stay at 90°.

- There is a carnival sideshow named the "wall of death" or "well of death" where motorcyclists travel along barrel-shaped walls along the vertical inner walls.

Quick rederivation of the minimum speed formula for the "well of death":

\[ mg = F_p \leq \mu_s N = \mu_s \frac{mv^2}{R} \Rightarrow v^2 \geq \frac{gR}{\mu_s} \]

It is worthy to note that banking is also the way airplanes turn in air. We can think of the lift (L) as the "normal" that air effectively applies on the plane. The vertical component of the lift opposes mg & the horizontal component allows the plane to turn.

- As the plane turns, the pilot (if untrained) has the tendency to bend her upper body to align "upwards", i.e., in the -\( \beta \) direction. The Airplane Flying Handbook by the U.S. Federal Aviation Agency states that such a bending posture is wrong. Your body should stay vertical to the floor of the plane.

Question: Why?

A related question: Imagine a person driving the motorcycle in the "well of death". What would she feel? How is it that she doesn't fall off the motorcycle?